Module-I

Quantum Mechanics

Mechanics: the study of the behavior of physical bodies when subjected to forces or displacements

Classical Mechanics: describing the motion of macroscopic objects. Macroscopic: measurable or observable by naked eyes

Quantum Mechanics: describing behavior of systems at Microscopic level (atomic length scales and smaller)

Classical/Newtonian Mechanics:

- Describes the dynamics of moving objects at macroscopic level.
- Applicable to all objects which have considerable mass i.e., sand stone to asteroids and spherical shell to massive planets.
- Fails to explain phenomena at microscopic level.

Quantum Mechanics:

- Describes the dynamics of moving objects at microscopic level.
- Applicable to particles with negligible mass such as subatomic particles (electron, proton, neutron) and photons, phonons.
- Explains all phenomena where CM fails.

Failures of Classical/Newtonian Mechanics:

Classical mechanics fails to explain following phenomena based on its assumptions that energy emitted or absorbed by a body is continuous and light is a form of wave.

1. Blackbody spectrum
2. Atomic spectrum of hydrogen atom
3. Photoelectric effect and Compton effect etc.,

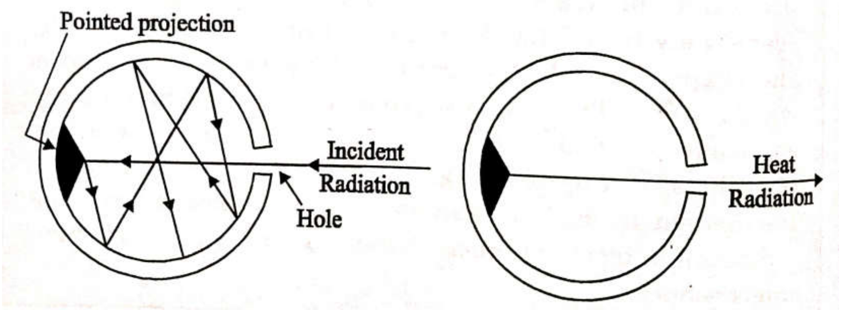
Black body radiation

A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.

Blackbody radiation does not depend on the type of object emitting it. Entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

Black body: A hollow spherical shell whose inner surface coated with lamp black acts as a black body.

- A fine hole is made and a pointed projection is provided just in front of the hole.
- When the radiations enter into this spherical shell through the hole, the heat radiations suffer multiple reflections and they are completely absorbed. Now this body acts as an absorber.
- When this object is heated to higher temperatures, it emits radiation through the hole. This radiation is known as ***Blackbody radiation***.



Perfect Blackbody: A perfect blackbody is one which absorbs all the heat radiations (all the wavelengths) incident on it. Further, when such a body is placed at constant high temperature, it emits radiation of all wavelengths.

Blackbody radiation: The heat radiation emitted from a blackbody is known as blackbody radiation.

The wavelength at which the maximum energy of radiation emitted depends only on temperature of the blackbody and it does not depend on the nature of the material.

Laws of Blackbody Radiation:

Wien's displacement Law: This law states that the product of wavelength (λ_m) corresponding to the maximum energy of radiation and absolute temperature of the blackbody (T) is a constant.

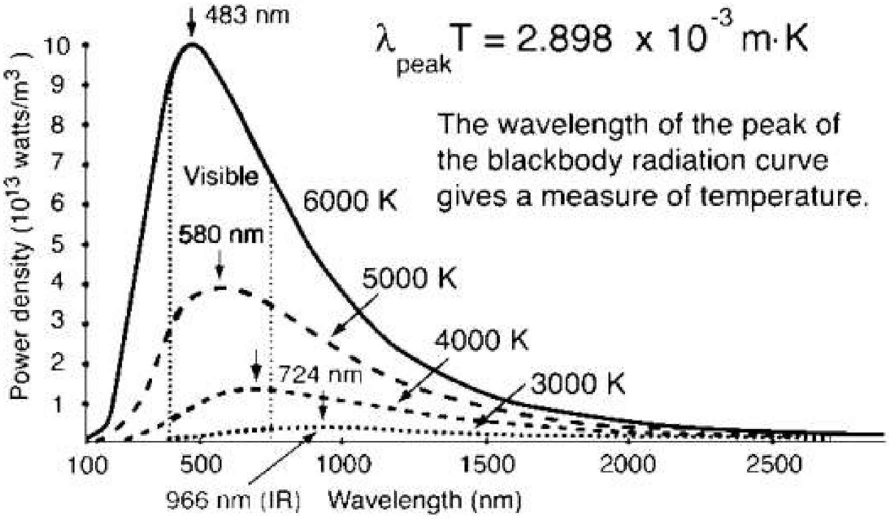
$$\text{i.e.,}\quad \lambda_m T = \textbf{constant}.$$

$$\lambda_m = \text{constant} / T$$

$$\text{or}\quad \lambda_m \propto 1/T$$

From above equation, it can be observed that the wavelength corresponding to the maximum energy of the blackbody radiation is inversely proportional to absolute temperature.

As the temperature of the blackbody increases, the wavelength corresponding to maximum energy decreases. Constant of proportionality is called Wien's displacement constant and equals **2.897768×10^{-3} m K**



The spectral radiance of blackbody radiation shows that:

1. The higher is the temperature, the more the energy emission and the shorter is the average wavelength. ($E \alpha 1/\lambda$ and $T \alpha 1/\lambda$ i.e., $E \alpha T$)
2. Less amount of energy emitted at very low wavelength.
3. The power radiation increases rapidly as λ increases from very small value.
4. The power radiation is most intense at certain wavelength λmax or υmax for particular temperature.
5. λ_m decreases linearly with increasing temperature.

Wien's Radiation Law: Wien deduced a law for the energy emitted by a blackbody at a given wavelength (λ) and temperature (T) to explain the blackbody spectrum. It is known as Wien's radiation law.

The energy density in the wavelength range λ and $\lambda + d\lambda$ is given by

$$E_\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{h\nu/kT}} = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}} \qquad \left[\because \nu = \frac{c}{\lambda}\right]$$

$$E_\lambda = 8\pi hc\lambda^{-5} e^{-hc/\lambda kT}$$

$$\boxed{E_\lambda = C_1 \lambda^{-5} e^{-C_2/\lambda T}}$$

where C_1 and C_2 are constants

T is the temperature of the blackbody, and C_1 and C_2 are constants whose values are given by

$$C_1 = 8\pi hc \quad \text{and} \quad C_2 = hc/k$$

Limitation: This law holds good only for shorter wavelengths and not for longer wavelengths.

Rayleigh – Jean's Law

This law states that the energy distribution of a black body is directly proportional to the absolute temperature (T) and inversely proportional to the fourth power of the wavelength (λ).

$$E_\lambda \, \alpha \, T \qquad \text{and} \qquad E_\lambda \, \alpha \, \frac{1}{\lambda^4}$$

$$E_\lambda \, \alpha \, \frac{T}{\lambda^4}$$

$$E_\lambda \, \alpha \, \frac{8\pi kT}{\lambda^4}$$

Where k is the Boltzmann constant. $k = 1.38064852 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}

Limitation: This law holds good only for longer wavelengths and not for shorter wavelengths.

Planck's theory

Max Planck proposed quantum theory around 1900, based on which he succeeded in explaining the blackbody radiation at all wavelengths. According to Max Planck,

- **A black body** is not only filled up with the radiations but also with a large number of tiny oscillators. They are of atomic dimensions. Hence they are known as atomic oscillators or Planck's oscillators.
- Each of these oscillators is vibrating with a characteristic frequency.
- The frequency of radiation emitted by oscillator is same as that of oscillator frequency.
- The oscillator cannot absorb or emit energy in a continuous manner. It can absorb or emit energy in multiples of small units called *quantum*.
- This quantum of radiation is called photon. The energy of the photon (E) is directly proportional to the frequency of radiation (ν)

$$\varepsilon \; \alpha \; \nu$$

$$\varepsilon = h \nu$$

h is Planck's constant. **$h = 6.6 \times 10^{-34}$ Js**

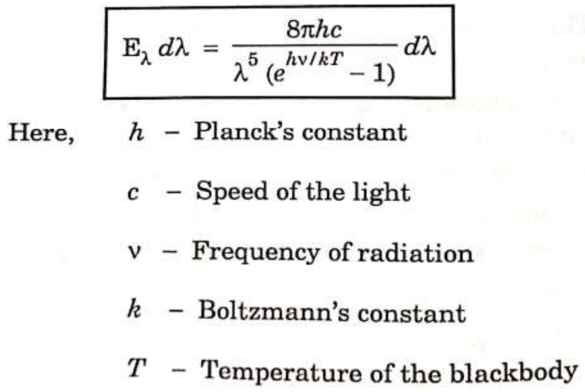
The oscillator vibrating with frequency ν can only emit energy in quantum of values $h\nu$. It indicates that the oscillators vibrating with frequency ν can only have discrete energy values E_n.

It is given by $E_n = n h \nu = n \varepsilon$

Where n is a positive integer i.e., 1,2,3 ……..

It means that the energy of the atomic oscillator is quantized and integer n is known as quantum number.

Planck's Radiation Law: According to Planck, the energy density of heat radiation emitted from a blackbody at temperature T in the wavelength range from λ to $\lambda + d\lambda$ is given by



$$\boxed{E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5 \left(e^{h\nu/kT} - 1\right)} d\lambda}$$

Here, h – Planck's constant

c – Speed of the light

ν – Frequency of radiation

k – Boltzmann's constant

T – Temperature of the blackbody

The Stefan-Boltzmann Law

According to Stefan-Boltzmann law, the amount of energy radiated is proportional to the temperature of the object raised to the fourth power.

$$\mathbf{E \; \alpha \; T^4}$$

➡ The Stefan Boltzmann equation

$$\mathbf{E = \sigma T^4}$$

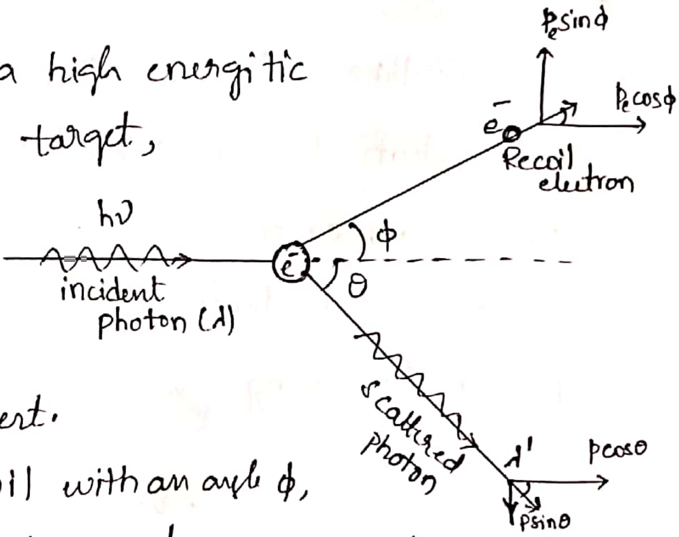
E = Energy density (W/m^2), T = temperature (K),**$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4** (Stefan-Boltzmann constant)

This law gives the total energy being emitted at all wavelengths by the blackbody (which is the area under the Planck Law curve)

Compton Effect :-

When high energitic, monochromatic x-rays of wavelength 'λ' ~~incident on~~ strikes a material target, the scattered beam consists of two radiations of wavelength λ and λ', i.e, one is having wavelength same as that of incident and the other beam have slightly greater wavelength than that of incident. This phenomenon is known as compton effect, and the difference in wavelength $\Delta\lambda = \lambda' - \lambda$ is called compton shift.

According to Compton, when a high energitic X-ray photon strikes the metal target, it transfers a part of its energy ($h\nu$) to the loosely bound electron in the metal. This electron is considered to be at rest.



After collision the electron recoil with an angle ϕ, and the scattered photon with decreased energy ($h\nu'$) which have longer wavelength(λ') than incident, travel making an angle 'θ' with incident direction.

Here, the collision between photon and electron is considered to be perfectly elastic collision. Hence, it obeys both law of conservation of energy and law of conservation of momentum.

Before Collision

- Energy of incident photon = $h\nu$

Momentum of incident photon = $\frac{h\nu}{c}$ $\left[\because E = mc^2,\ \frac{E}{c} = mc \Rightarrow P = \frac{h\nu}{c}\right]$

Energy of electron at rest = m_0c^2 [∵ m_0 rest mass of e^-]

momentum of e^- at rest = 0 $\longrightarrow$ ①

After collision

Energy of scattered photon = $h\nu'$

momentum of scattered photon = $\frac{h\nu'}{c}$ $\longrightarrow$ ②

Energy of recoil electron = mc^2

momentum of recoil electron = mv

Applying law of conservation of energy, we have

Total energy before collision = Total energy after collision

$$\Rightarrow h\nu + m_0c^2 = h\nu' + mc^2$$

$$\Rightarrow mc^2 = h(\nu - \nu') + m_0c^2 \longrightarrow ③$$

Applying law of conservation of momentum, in the direction of incidence we have

Momentum before collision = Momentum after collision

$$\Rightarrow \frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$\Rightarrow mvc\cos\phi = h\nu - h\nu'\cos\theta \longrightarrow ④$$

Applying law of conservation of momentum $\perp$r to direction of incidence

$$\Rightarrow 0 + 0 = -\frac{h\nu'}{c}\sin\theta + mv\sin\phi$$

$$\Rightarrow mvc\sin\phi = +h\nu'\sin\theta \longrightarrow ⑤$$

Squaring and adding equ ④ + ⑤

$$\Rightarrow m^2v^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

$$\Rightarrow m^2v^2c^2 = h^2[\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta + \nu'^2\sin^2\theta]$$

$$\Rightarrow m^2v^2c^2 = h^2[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta] \longrightarrow ⑥$$

Squaring eq.③ on both sides, we get

$$m^2c^4 = h^2(\nu-\nu')^2 + m_0^2c^4 + 2h(\nu-\nu')\,m_0c^2$$

$$m^2c^4 = h^2(\nu^2+\nu'^2-2\nu\nu') + m_0^2c^4 + 2hm_0c^2(\nu-\nu') \rightarrow ⑦$$

Subtracting equ. ⑥ from ⑦, we get

$$m^2c^4 - m^2c^2v^2 = h^2(\nu^2+\nu'^2-2\nu\nu') + m_0^2c^4 + 2hm_0c^2(\nu-\nu') - h^2(\nu^2+\nu'^2-2\nu\nu'\cos\theta)$$

$$\Rightarrow \quad m^2c^2(c^2-v^2) = m_0^2c^4 + 2hm_0c^2(\nu-\nu') - 2h^2\nu\nu'(1-\cos\theta)$$

According to theory of relativity $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$

$$\therefore \quad \frac{m_0^2c^2}{(1-v^2/c^2)}(c^2-v^2) = m_0^2c^4 + 2hm_0c^2(\nu-\nu') - 2h^2\nu\nu'(1-\cos\theta)$$

$$\Rightarrow \quad \frac{m_0^2c^4}{(c^2-v^2)}(c^2-v^2) = m_0^2c^4 + 2hm_0c^2(\nu-\nu') - 2h^2\nu\nu'(1-\cos\theta)$$

$$\Rightarrow \quad 2hm_0c^2(\nu-\nu') = 2h^2\nu\nu'(1-\cos\theta)$$

$$\Rightarrow \quad \frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\Rightarrow \quad \left(\frac{1}{\nu'}-\frac{1}{\nu}\right) = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\left[\because c = \nu\lambda \Rightarrow \frac{1}{\nu} = \frac{\lambda}{c}\right]$$

$$\Rightarrow \quad \lambda'-\lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Rightarrow \quad \boxed{\Delta\lambda = \lambda'-\lambda = \frac{h}{m_0c}(1-\cos\theta)} \rightarrow ⑧$$

$\Delta\lambda$ is called as Compton wavelength shift. From equ.⑧, it is clear that the wavelength shift directly depends on the scattering angle 'θ'.

Case (i) when $\theta = 0 \Rightarrow \cos\theta = 1$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = 0$$

$$\therefore \lambda' - \lambda = 0 \Rightarrow \boxed{\lambda' = \lambda}$$

$\therefore$ No wavelength shift is observed in the direction of incidence.

Case (ii) $\theta = 90° \Rightarrow \cos\theta = 0$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1)$$

$$\therefore \Delta\lambda = \frac{h}{m_0 c} = \frac{6.6\times10^{-34}}{9\times10^{-31}\times3\times10^{8}} = 0.024\,\mathring{A}$$

$$\therefore \boxed{\lambda_c = \frac{h}{m_0 c} = 0.024\,\mathring{A}}$$

'λ_c' is called as Compton wavelength.

Case (iii) : $\theta = 180° \Rightarrow \cos\theta = -1$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(2) = \frac{2h}{m_0 c} = 2\lambda_c$$

$$\boxed{\Delta\lambda = 2\lambda_c = 0.048\,\mathring{A}}$$

In this case the wavelength shift is twice the Compton wavelength.

Wave-particle duality :-

It is clear from photoelectric effect and Compton effect that light (or) radiation also posses particle behaviour in addition to wave nature. Similarly, matter is also expected to behave like wave in addition to its particle nature in certain conditions. This proposal was put forward by de Broglie in 1924.

This is known as dual nature (or) wave-particle duality.

de Broglie hypothesis & Matter waves

According to de Broglie, matter also posses dual nature, like radiation. Every material object with mass 'm' and moving with velocity 'v' is associated with a wave, known as matter waves.

According to Planck's theory of radiation, the energy of a photon

$$E = h\nu = \frac{hc}{\lambda} \rightarrow ①$$

where c is velocity of light and λ is wavelength.

According to Einstein mass-energy relation,

$$E = mc^2 \rightarrow ②$$

from equ ① & ② $\Rightarrow \frac{hc}{\lambda} = mc^2$

$$\Rightarrow \boxed{\lambda = \frac{h}{mc} = \frac{h}{p}} \rightarrow ③$$

where p = mc is momentum of photon.

Equation ③ is also applicable for all material particles with mass 'm' and velocity 'v' and having momentum p = mv.

$$\therefore \boxed{\lambda = \frac{h}{p} = \frac{h}{mv}} \rightarrow ④$$

Equ. ④ is known as de Broglie wavelength. The wavelength can be expressed interms of energy as follows

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}} \rightarrow ⑤ \qquad \because \left[E = \frac{1}{2}mv^2 = \frac{p^2}{2m}\right]$$

de-Broglie wavelength associated with electrons

Consider an electron of rest mass m_0, charge 'e' and accelerated by a potential 'V' volts is moving with velocity 'v' then

$$\text{kinetic energy } E = \frac{1}{2} m_0 v^2 = eV$$

$$(\text{or}) \quad v = \sqrt{\frac{2eV}{m_0}}$$

$$\text{de Broglie wavelength } \lambda = \frac{h}{m_0 v} = \frac{h}{m_0}\sqrt{\frac{m_0}{2eV}}$$

$$\lambda = \frac{h}{\sqrt{2eVm_0}}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.632 \times 10^{-19} \times V \times 9.1 \times 10^{-34}}}$$

$$\therefore \boxed{\lambda = \frac{12.26}{\sqrt{V}} \mathring{A}}$$

Above equ. represents the de Broglie wavelength for electrons.

If an electron is accelerated with V = 100 volts, then

$$\boxed{\lambda = \frac{12.26}{\sqrt{100}} = 1.226 \, \mathring{A}}$$

Characteristics of Matter Waves

The de Broglie wavelength for Matter waves is given by

$$\lambda = \frac{h}{mv}$$

1) Since $\lambda \propto \frac{1}{m}$, lighter the particle, greater is the wavelength associated with it.

2) Since $\lambda \propto \frac{1}{v}$, lesser is the velocity of particle, greater is the wavelength

3) for $v = 0$, $\lambda = \infty$. Only moving objects can have matter waves.

4) The de Broglie wavelength does not depend on charge of the particle. Hence there are not electromagnetic waves.

5) The velocity of matterwaves is greater than light velocity.

we know that $E = h\nu$ and $E = mc^2$

$$h\nu = mc^2$$

$$\nu = \frac{mc^2}{h}$$

velocity of matter waves $\omega = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{p}$

$$\omega = \frac{mc^2}{h} \frac{h}{mv}$$

$$\boxed{\omega = \frac{c^2}{v}}$$

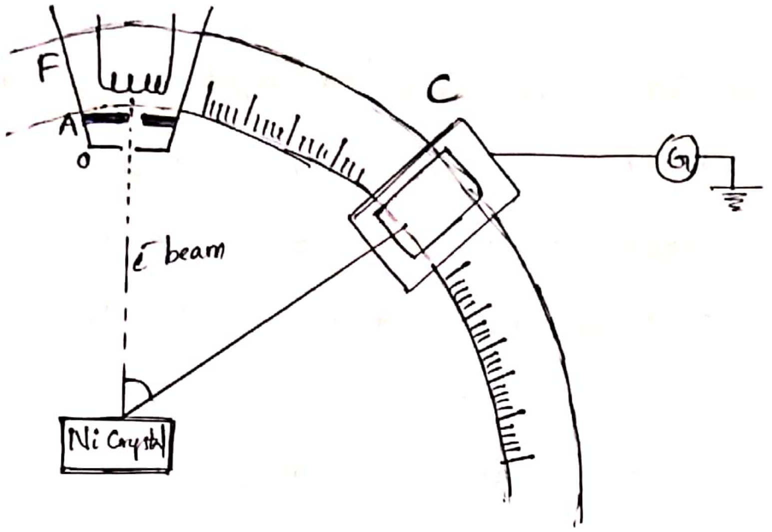
Davisson and Germer's Experiment

Davisson and Germer performed electron diffraction experiment, which is a strong evidence for existance of matter waves proposed by de Broglie.

Principle:- As suggested by de-Broglie, every moving particle is associated with a wave called matter wave. In this experiment fast moving electrons are diffracted using a Nickel crystal. Since diffraction is a wave phenomenon, this confirms matter also behaves like waves and have dual nature.

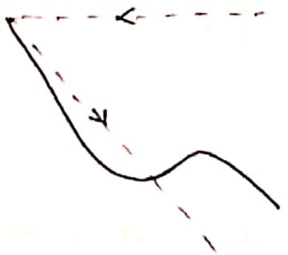
Experimental details

The experimental arrangement used for electron diffraction is as shown in figure. It consists of a hot filament 'F' which emits electron beam. This electron beam is accelerated by maintaining a potential difference between 'F' and anode 'A'. The electron beam emerges from a fine aperture 'O' and incident on a Nickel crystal 'N'.



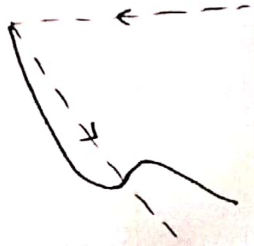
The electrons, scattered from 'Ni' crystal are collected by a faraday cylinder 'C', to measure the intensity of scattered electrons by measuring the current using a galvanometer 'G' connected to it. The faraday cylinder is fixed on a semicircular ~~Working~~:- scale to collect the scattered electrons at all possible deflections between the angles 20° to 90°. The accelerating potential can be varied in the range 30 to 600 V.

Working:- The accelerating potential is slowly increased and at each potential, the current is measured as a function of scattering angle by moving the faraday cylinder. The atomic planes in the 'Ni' crystal acts as diffraction grating and diffracts the incoming electron beam in different orders. Curves were plotted between scattering angle and current for each applied voltage.

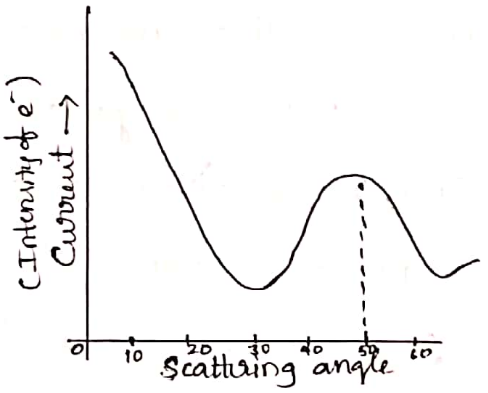

44 Volts

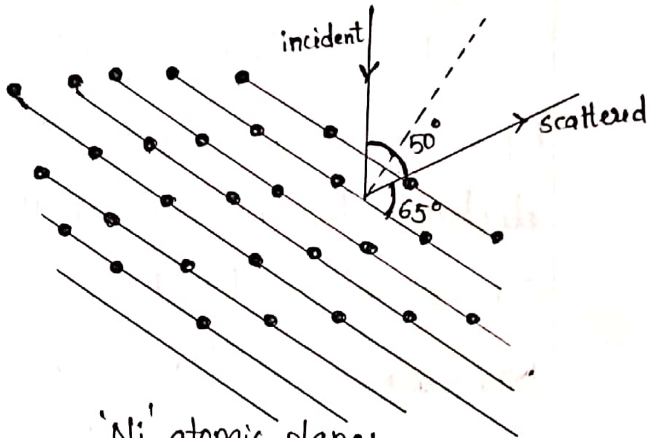

48 Volts

54 Volts


68 Volts.

The Curves remained smooth until a voltage of 44 volts, after which a small hump is observed. The hump increased with increasing voltage and became maximum at 54 volts for a scattering angle of 50°. After this, the hump decreases with increase in voltage and vanished at 68 Volts. The appearance of a large hump at 54 volts is a strong evidence of electrons being diffracted.





'Ni' atomic planes

The wavelength of diffracted electrons can be calculated similar to X-ray diffraction phenomenon using the formula.

$$2d\sin\theta = n\lambda$$

The interatomic spacing for 'Ni' crystal $d = 0.91\,\mathring{A}$ and the glancing angle $\theta = 65°$.

$$\Rightarrow 2(0.91)\sin 65° = \lambda \qquad [\because n=1, \text{first order}]$$

$$\Rightarrow \boxed{\lambda = 1.65\,\mathring{A}}$$

From de Broglie hypothesis we have

$$\lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{54}} = 1.66\,\mathring{A}$$

$$\Rightarrow \boxed{\lambda = 1.66\,\mathring{A}}$$

The experimental value of wavelength of e^- is closely matched with the theoritical value proposed by de Broglie. This Confirms the existance of Matter waves.

GP Thomson experiment:

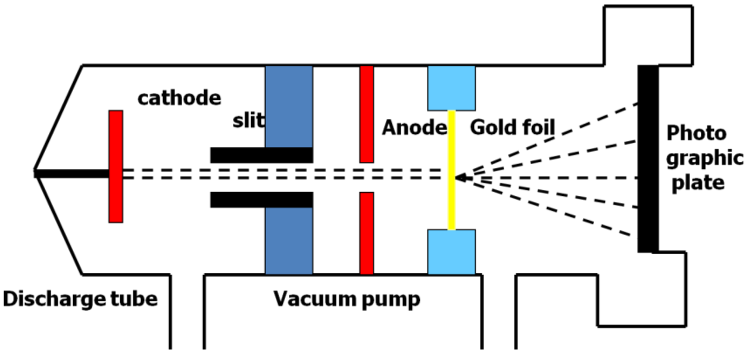
G.P Thomson, in 1928 performed an experiment with high kinetic energy electrons using thin gold foil as the target material and observed the diffraction pattern due to scattered electrons by gold foil. This experiment proves the wave nature of electrons and hence the de Broglie hypothesis of matter waves.

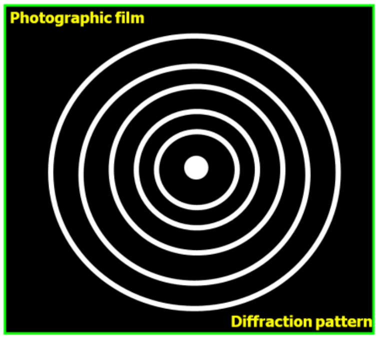
Principle:

The fast moving electrons are scattered by the gold foil and produced diffraction pattern on the screen. Since diffraction is a wave phenomenon, it proves that electrons are behaving as waves. This is the proof of dual nature of matter as proposed by de Broglie.

Experimental arrangement:

The experimental arrangement of G.P Thomson electron diffraction is as shown in figure. It consists of a filament or cathode C which emits the electrons. A narrow slit S is used to collimate the electron beam emitted from cathode. Potential difference of 10 – 50 KV is applied between cathode and anode to accelerate the electrons to very high kinetic energies. These high KE electrons are made to strike a thin gold foil of thickness 100 A^{o} (10^{-8}m) and get scattered. The scattered electrons are captured on a photographic plate kept next to gold foil. The whole setup is kept in a highly evacuated chamber (If not the electrons lose KE by colliding with molecules of gas/ dust particles).





Working:

- ✓ Using a suitable battery the cathode can be heated, so that electrons will be emitted and pass through a high positive potential given to the anode ‘A’, then the electron beam passes through a fine slit and incidents on the gold foil of thickness 10^{-8} m.

- ✓ The electrons passing through the gold foil are recorded on a photographic plate. Since the gold foil consists a large number of microscopic crystals oriented at random, the electrons striking the gold foil diffracts according to Bragg’s law 2d sinθ = nλ.

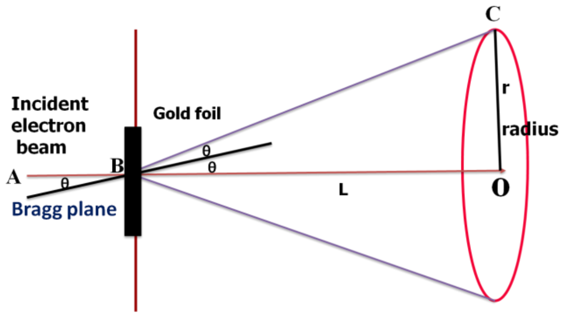
- ✓ After developing the plate, a symmetrical pattern consisting of concentric rings about a central spot is obtained. This pattern is as similar as the pattern produced by X-rays generated by the high KE electrons.

- ✓ To confirm that the pattern produced is due to electrons but not due to the X-rays, magnetic field is applied normally in between the gold foil and the screen. The pattern is observed to be shifted on application of the magnetic field.

- ✓ This confirms that the pattern is produced by electrons and not by X-rays (X-rays pattern is not affected by electric and magnetic fields). Hence the pattern is formed due to diffraction of electrons by the crystal planes of gold foil.
- ✓ As the diffraction pattern can only be produced by waves and not by the particles. So Thomson concluded that electrons behave like waves.

Theory:

From the figure, OC is the radius ‘r’ of the ring as O is the center of the ring, B is the point where the electrons strikes a particular point on the gold foil. BO is the distance between the gold foil and the photographic plate and is represented by L.



From figure tan 2θ = r / L

⇨ If θ is very small, tan (θ) ≈ θ

⇨ Hence 2θ = r / L ……………. (1)

From Bragg’s law, 2d sinθ = nλ

⇨ If θ is very small, sin (θ) ≈ θ

⇨ 2dθ = nλ

For first order diffraction n=1

⇨ 2θ = λ / d ……………………. (2)

From equ (1) and (2), we have r / L = λ / d

⇨ d = λ L / r …………………... (3)

From de Broglie wavelength for electrons, we have $\lambda = \frac{h}{\sqrt{2m_0 eV}}$ ………….. (4)

Here, m_0 is the rest mass of electron.

Substitute equ (4) in equ (3) ➔ $d = \frac{L}{r}(\frac{h}{\sqrt{2m_0 eV}})$

⇨ $d = 4.08^0 A$

From X-ray diffraction method for first order diffraction, $2d\sin\theta = n\lambda$

⇨ d = λ / (2 sinθ) = 4.06 Å (n=1)

Thus the value of ‘d’ obtained from G.P. Thomson experiment and X-ray diffraction method are the same, demonstrating the de Broglie’s concept of matter waves. Thomson also concluded that the wavelength of electron only depends on the accelerating voltage and is independent of the material of the target.

Heisenberg's uncertainity Principle

In quantum mechanics a particle can be treated as a group of waves called as 'wave packet'. If wave packet is small, then the position (x) of the particle can be precisely measured but not the momentum (p). If the wave packet is large, then pos it is difficult to precisely find the position but momentum can be measured.

According to Heisenberg 'It is not possible to measure precisely and simultaneously both the position and momentum (p) of a micro particle'. If Δx and ΔP_x be the uncertainities in measuring position & momentum respectively, then

$$\boxed{\Delta x \, \Delta P_x \geq \frac{h}{4\pi}} \longrightarrow ①$$

This is known as Heisenberg's uncertainity principle. This is a direct consequence of wave nature of particles which is based on de Broglie's hypothesis of matter waves.

In addition to position & momentum, the other pairs are Energy and time, and angular displacement and angular momentum.

$$\therefore \boxed{\Delta E \, \Delta t \geq \frac{h}{4\pi}} \quad \text{and} \quad \boxed{\Delta \theta \, \Delta L \geq \frac{h}{4\pi}}$$

Consequences (or) Applications of Heisenberg's Uncertainity Principle

1. Non Existance of electrons inside the nucleus

Using Uncertainity principle we can confirm that electrons can not present inside nucleus.

We know that radius of nucleus is 10^{-14} m. If electron exist inside nucleus, then it must be found within a distance of 2×10^{-14} m.

∴ The uncertainity in position $\Delta x = 2\times10^{-14}$ m.

According to uncertainity principle, $\Delta x \Delta p \geq \frac{h}{4\pi}$

uncertainity in momentum $\Delta p \geq \frac{h}{4\pi\Delta x}$

$$\Rightarrow \Delta p \geq \frac{6.63\times10^{-34}}{4(3.14)(2\times10^{-14})}$$

$$\Rightarrow \Delta p \geq 0.5\times10^{-20}\ \text{kg.m/sec.}$$

To have such large momentum value, the electron must be travelling with velocity of light.

∴ Energy of electron $E = mc^2$

$$\Rightarrow E = (mc)c$$

$$\Rightarrow E = pc$$

$$\Rightarrow E = (0.5\times10^{-20})(3\times10^{8})$$

$$\Rightarrow E = 1.5\times10^{-12}\ J$$

(or) $E \approx 100$ MeV

If an electron exist inside nucleus, it must have energies of the order 100 MeV. It is observed in β-decay, the electrons emitted have energies less than 5 MeV. This confirms that electrons can not exist inside nucleus.

Energy of an electron in an atom

~~Consider Hydrogen atom, which consists only one electron. According to Bohr, the radius of first orbit is 5.3×10^{-11} m.~~

~~According to uncertainity principle $\Delta p \geq \frac{h}{4\pi(\Delta x)}$~~

~~Since $\Delta x = 5.3\times10^{-11}$ m~~

~~$\Rightarrow \Delta p \geq \frac{h}{4(3.14)(5.3\times10^{-11})}$~~

~~$$\Delta P \gtrsim 0.1 \times 10^{-23} \text{ kg·m/sec.}$$~~

~~$\therefore$ Kinetic energy of electron $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$~~

~~$$\Rightarrow E = \frac{(0.1 \times 10^{-23})^2}{2(9.1 \times 10^{-31})}$$~~

~~$$\Rightarrow E \simeq 3.44 \text{ eV}$$~~

~~This energy is less than the actual energy of electron in first orbit i.e., 13.6eV. Hence, the position & momentum can't be measured simultaneously and accurately.~~

Schrodinger Wave Equation

Based on planck's quantum theory and concept of de Broglies matter waves, schrodinger proposed a generalized wave equation which can be used to estimate the possible energy levels of a free electron.

Don't Write

In wave mechanics, the wave associated with a moving particle is represented using a wave function as follows

$$\psi(x,t) = A \exp(-i(\omega t - kx)) \longrightarrow ①$$

where $A \to$ amplitude, $\omega \to$ angular freq $(2\pi\nu)$ and $K \to$ propagation constant $(2\pi/\lambda)$

$$\Rightarrow \psi(x,t) = A \exp\left(-2\pi i\left(\nu t - \frac{x}{\lambda}\right)\right)$$

We know that $E = h\nu$ and $\lambda = \frac{h}{p}$

$$\Rightarrow \psi(x,t) = A \exp\left(-2\pi i\left(\frac{E}{h}t - \frac{px}{h}\right)\right)$$

$$\Rightarrow \psi(x,t) = A \exp\left(-\frac{i}{\hbar}(Et - px)\right) \longrightarrow ② \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

The total energy of a free electron $E = KE + PE$

$$E = \frac{1}{2}mv^2 + V$$

$$E = \frac{p^2}{2m} + V$$

Hence $E\psi = \frac{p^2}{2m}\psi + V\psi \longrightarrow ③$

Differentiating equ. ② w.r.to time, we get

$$\frac{\partial\psi}{\partial t} = \left(\frac{iE}{\hbar}\right) A\exp\left(-(i/\hbar)(Et - px)\right)$$

$$\frac{\partial\psi}{\partial t} = i\hbar E\psi$$

$\therefore E\psi/t$

$$\Rightarrow \frac{\partial\psi}{\partial t} = \left(-\frac{iE}{\hbar}\right) A\exp\left(-\frac{i}{\hbar}(Et - px)\right)$$

$$\Rightarrow \frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi$$

$$\Rightarrow E\psi = i\hbar\frac{\partial\psi}{\partial t} \longrightarrow ④$$

Differentiating equ. ② twice w.r.to 'x', we get

$$\frac{\partial\psi}{\partial x} = \left(\frac{ip}{\hbar}\right) A\exp\left(-\frac{i}{\hbar}(Et - px)\right)$$

$$\Rightarrow \frac{\partial\psi}{\partial x} = \frac{ip}{\hbar}\psi$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)\left(\frac{ip}{\hbar}\right)\psi$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi$$

$$\Rightarrow p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \longrightarrow ⑤$$

Substituting equ ④ & ⑤ in equ ③, we get

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi} \rightarrow ⑥$$

The above equation represents schrodinger (time-dependent) wave equation in one-dimension. Equ ⑥ for three dimension can be written as

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi} \rightarrow ⑦ \left[\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$$

Schrodinger time-independent wave equation:-

In general, it is convinient to consider the potential energy of the particle depends only on space but not time. In such case, the schrodinger time-dependent wave equ. reduces to time-independent wave equation.

we know that $\psi(x,t) = A\exp\left[\frac{-i}{\hbar}(Et - px)\right]$

$$\psi(x,t) = A\exp\left(\frac{-iEt}{\hbar}\right)\exp\left(\frac{ipx}{\hbar}\right)$$

$$\psi(x,t) = A\phi(t)\psi(x) \rightarrow ①$$

where $\psi(x) = \exp\left(\frac{ipx}{\hbar}\right)$ and $\phi(t) = \exp\left(\frac{-iEt}{\hbar}\right)$

$$\therefore \quad \psi(x,t) = A\psi\exp\left(\frac{-iEt}{\hbar}\right)$$

differentiating w.r.t time $\Rightarrow \frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar}A\psi\exp\left(\frac{-iEt}{\hbar}\right) \rightarrow ②$

differentiating w.r.t x $\Rightarrow \frac{\partial \psi}{\partial x} = A\exp\left(\frac{-iEt}{\hbar}\right)\frac{\partial \psi}{\partial x}$

and $\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = A\exp\left(\frac{-iEt}{\hbar}\right)\frac{\partial^2 \psi}{\partial x^2} \rightarrow ③$

Substituting equ ② & ③ in schrodinger time dependent wave equ.

i.e., $i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$

~~$\Rightarrow i\hbar\left(-\frac{iE}{\hbar}A\psi\exp\left(-\frac{iEt}{\hbar}\right)\right) = -\frac{\hbar^2}{2m}\left[A\exp\left(-\frac{iEt}{\hbar}\right)\frac{\partial^2\psi}{\partial x^2}\right] + V\psi$~~

~~$\Rightarrow E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \rightarrow ④$~~

~~Above equation represents schrodinger time-independent wave equation. Equ. ④ can also be written as~~

~~1D $\Rightarrow \boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \rightarrow ⑤$~~

~~3D $\Rightarrow \boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \rightarrow ⑥$~~

* Physical Significance and Born's interpretation of the wave function 'ψ':-

1. The wavefunction 'ψ' has no direct physical meaning. It represents the variation of matterwaves.

2. The wavefunction contains all the necessary information of a system. And the information can be obtained using various operators such as momentum, energy etc..,

3. According to Born's interpretation, $\psi.\psi^*$ (or) $|\psi|^2$ is the probability density function, i.e., the probability of finding the particle.

$$\boxed{\int_{-\infty}^{\infty}\psi\psi^*\,dx\,dy\,dz = 1}$$

* ψ^* is Complex Conjugate of ψ.

This is called as Normalization Condition.

✱ Time independent Schrodinger wave Equation
(Alternate treatement)

Based on planck's quantum theory and de Broglie hypothesis of matter waves, Schrodinger proposed a generalized wave equation for matter waves which is useful in estimating the energy levels of electrons in atoms / crystals.

In wave mechanics, the wave associated with a moving particle is represented using a wave function as follows.

$$\psi = \psi_0 \sin \omega t \longrightarrow ①$$

where $\psi_0 \to$ amplitude and $\omega = 2\pi\nu$ is the angular frequency.

$$\Rightarrow \psi = \psi_0 \sin 2\pi\nu t$$

differentiating above equ. w.r.to time, twice, we get

$$\frac{\partial\psi}{\partial t} = \psi_0 (2\pi\nu) \cos 2\pi\nu t$$

and $$\frac{\partial^2\psi}{\partial t^2} = -\psi_0 (2\pi\nu)^2 \sin 2\pi\nu t$$

$$\Rightarrow \frac{\partial^2\psi}{\partial t^2} = -4\pi^2\nu^2\psi \longrightarrow ②$$

The generalized wave equation is given by

$$\frac{\partial^2\psi}{\partial t^2} = v^2 \frac{\partial^2\psi}{\partial x^2} \longrightarrow ③$$

where 'v' represents velocity of the wave.

Comparing equ ② & ③, we get

$$v^2 \frac{\partial^2\psi}{\partial x^2} = -4\pi^2\nu^2\psi \qquad (\because v = \nu\lambda)$$

$$\Rightarrow \nu^2\lambda^2 \frac{\partial^2\psi}{\partial x^2} = -4\pi^2\nu^2\psi$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{-4\pi^2}{\lambda^2}\psi$$

from de Broglie hypothesis, we have $\lambda = \frac{h}{mv}$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{-4\pi^2 m^2 v^2}{h^2}\psi \rightarrow ④$$

we know that total energy $E = KE + PE$

$$\Rightarrow E = \frac{1}{2}mv^2 + V$$

$$\Rightarrow mv^2 = 2(E-V)$$

$$\Rightarrow m^2v^2 = 2m(E-V) \rightarrow ⑤$$

Substituting ⑤ in ①

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{-4\pi^2(2m(E-V))}{h^2}\psi$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-V)\psi \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$$\Rightarrow \boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0} \rightarrow ⑥$$

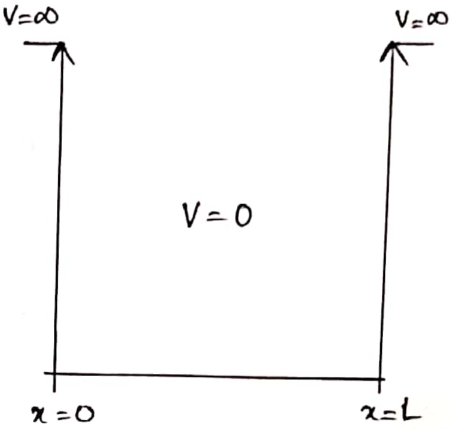
Above equ. is known as Schrodinger's time-independent wave equation in one dimension (x).

In 3 dimensions $\Rightarrow$ $\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$ $\left(\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

* Particle in one dimensional potential box:

Free electrons trapped in metal atoms by the +ve nuclear forces and ions in crystals can be considered as particles trapped in infinite potential well.

The one dimensional potential well is illustrated in the figure. within the well, the potential is zero and outside the well (box) potential is infinite.



$$\left.\begin{array}{ll} V(x)=0 & 0<x<L \\ V(x)=\infty & x\leq 0,\ x\geq L \end{array}\right\} \rightarrow ①$$

Since, the potential energy depends only on space, but not time i.e., V(x), to estimate the energies of trapped particle, Schrodinger time-independent equ. may be used.

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}E\psi=0 \qquad (\because V=0 \text{ inside box})$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2}+K^2\psi=0 \qquad \text{where } K^2=\frac{2mE}{\hbar^2} \longrightarrow ②$$

Above equ. is a second order differential equation, whose Solution will be of the form

$$\psi(x)=A\sin Kx+B\cos Kx \longrightarrow ③$$

where A, B are Constants.

Since the particle can not penetrate the walls, the wave function 'ψ' must be zero at walls. Hence, the boundary Conditions are

i) $\psi(x)=0$ at $x=0$ and

ii) $\psi(x)=0$ at $x=L$

Applying first boundary Condition

$$③ \Rightarrow 0=B$$

Applying second boundary condition

$$③ \Rightarrow 0=A\sin KL \longrightarrow ④ \quad (\because B=0)$$

$$\Rightarrow A\sin KL = 0$$

$$\Rightarrow \sin KL = 0$$

and $KL = n\pi$

$$\Rightarrow K = \frac{n\pi}{L}$$

$$\Rightarrow K^2 = \frac{n^2\pi^2}{L^2}$$

$$\Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$\Rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}} \quad (\text{or}) \quad \boxed{E = \frac{n^2h^2}{8mL^2}} \rightarrow ⑤ \quad \left(\because \hbar = \frac{h}{2\pi}\right)$$

Above equ. represents the energy of the particle trapped in one dimensional potential box. The energy is quantized and 'n' is the quantum number.

The wave function is given by

from equ ③ $\Rightarrow \psi(x) = A\sin Kx \qquad (\because B=0)$

Since, the particle is inside the box, the probability of finding the particle within the box from '0' to 'L' must be '1'.

$$\therefore \int_0^L |\psi(x)|^2 dx = 1$$

$$\Rightarrow \int_0^L A^2\sin^2 Kx \, dx = 1$$

$$\Rightarrow \frac{A^2}{2}\int_0^L [1 - \cos 2Kx]\, dx = 1$$

$$\Rightarrow \frac{A^2}{2}\left[x - \frac{\sin 2Kx}{2K}\right]_0^L = 1$$

$$\Rightarrow \frac{A^2}{2}\left[L - \frac{\sin 2KL}{2K}\right] = 1$$

$$\Rightarrow \frac{A^2}{2}\left[L - \frac{L}{2n\pi}\sin 2\left(\frac{n\pi}{L}\right)L\right] = 1 \quad \left[\because K = \frac{n\pi}{L}\right)$$

$$\Rightarrow \frac{A^2 L}{2} = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{L}}$$

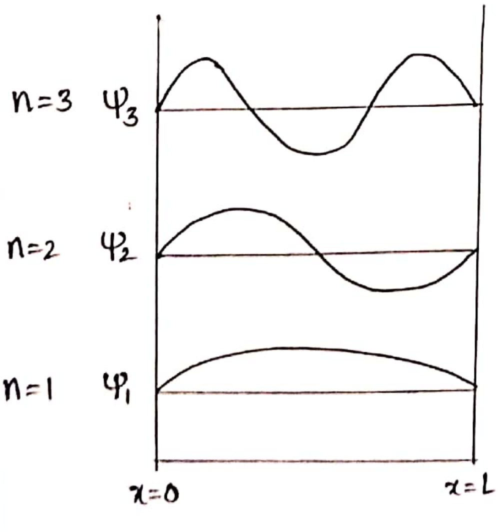
∴ The normalized wavefunction $\boxed{\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$

The wave functions (ψ_n) and Energies (E_n) corresponding to different 'n' values are called as eigen functions and eigen values, which describe the quantum state of the particle.

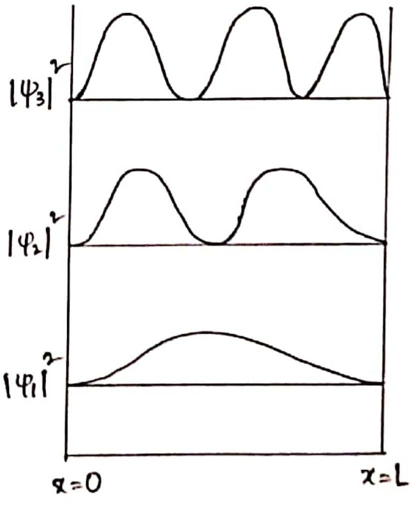
$$\psi_1 = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L} \quad \text{and} \quad |\psi_1|^2 = \frac{2}{L}\sin^2\frac{\pi x}{L}, \quad E_1 = \frac{h^2}{8mL^2}$$

$$\psi_2 = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L} \quad \text{and} \quad |\psi_2|^2 = \frac{2}{L}\sin^2\frac{2\pi x}{L}, \quad E_2 = \frac{4h^2}{8mL^2} = \frac{h^2}{2mL^2}$$

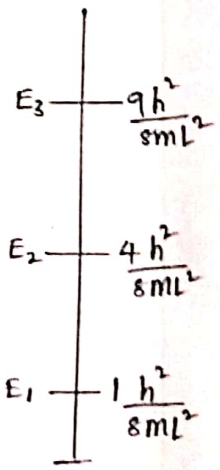
$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$



wave functions



probability densities



Energies

Module-II

FREE ELECTRON THEORY OF METALS

CLASSICAL FREE ELECTRON THEORY :-

Classical free electron theory obeys the laws of classical mechanics. Electron revolves around the nucleus.

→ The Valence electron of atoms are free to move about whole volume of metals like the molecules of a perfect gas in a Container. the free electrons obey the laws of the classical Kinetic theory of gases.

→ these free electrons move in random direction and Collide with either positive ions fixed to the lattice (or) other electrons: All the Collisions are elastic, i.e, there is no loss of energy.

→ The electron Velocities in a metal obey the classical Maxwell - Boltzmann distribution of velocities.

→ The free electrons move in a Completely uniform potential field due to ions fixed in the lattice.

→ When an electric field is applied to the metal, the free electrons are accelerated in the opposite to the direction of applied electric field.

SUCCESS OF CLASSICAL FREE ELECTRON THEORY :-

→ It verifies ohm's law.

→ It explains the electrical and the thermal Conductivities of metals.

→ It derives Wiedemann - Franz law. (i.e, the relation between electrical Conductivity and thermal Conductivity).

→ It explains the optical properties of metals.

DRAW BACKS OF CLASSICAL FREE ELECTRON THEORY

Inspite of the success seen above, classical theory has following drawbacks.

→ The phenomena such as photo electric effect, Compton effect and the black body radiation Could not be explained by classical free electron theory.

→ According to the classical free electron theory, the values of specific heat of metals is given by 4.5 Ru, where Ru is the universal gas Constant whereas the experimental value is nearly equal to 3Ru.

→ Also according to this theory the value of electronic specific heat is equal to $\frac{3}{2}$ Ru while the actual value is about 0.01 Ru only.

→ Though $\frac{K}{\sigma T}$ is a Constant (Wiedmann - Frenz Law). According to the classical free electron theory, but it is not a Constant at low temperature.

→ Ferromagnetism Could not be explained by this theory. The theoritical Value of paramagnetic susceptability is greater than experimental Value.

THE QUANTUM FREE ELECTRON THEORY :-

→ Sommerfeld proposed this theory in 1928 retaining the Concept of free electrons moving in a uniform potential within the metal as in the classical theory, but treated the electrons as obeying the laws of quantum mechanics.

→ Based on the de Broglie wave Concept, he assumed that a moving electron behaves as if it were a system of waves. (Called matter waves - waves associated with a moving particle).

→ According to quantum mechanics, the energy of an electron in a metal is quantized. The electrons are filled in a given energy level according to Pauli's exclusion principle. (i.e, No two electrons will have the same set of four quantum numbers).

→ Each energy level Can provide only two states namely, one with spin up and other with spin down and hence only two electrons Can be occupied in a given energy level.

→ It is assumed that the Valence electrons travel in Constant potential inside the metal but they are prevented from escaping the crystal by very high potential barriers at the ends of crystal.

→ In this theory, though the energy levels of the electron are discrete, the spacing between consecutive energy levels is very less and thus the distribution of energy levels seems to be continuous.

DRAWBACKS OF QUANTUM FREE ELECTRON THEORY :-

→ According to quantum mechanics, a conduction electron in a metal experiences constant (zero) potential and free to move inside the crystal but will not come out because an infinite potential exists at surface.

→ This theory explains electrical conductivity and thermal conductivity but failed to explain the difference between conductors, insulators and semiconductors.

→ It fails to explain positive hall coefficient of metals and low conductivities of divalent metals than monovalent metals.

Fermi-Dirac Statistics

- Fermi-Dirac statistics explains the behaviour of electron gas in metals.
- Electron gas consists of large no. of e^-, which are fermions and obeys Fermi-Dirac distribution function.
- According to Fermi-Dirac statistics, fermions are indistinguishable particles and obey pauli's exclusion principle.
- The electrons are distributed among various states in a three-dimensional box at thermal Equilibrium.

For a metal, considering a three dimensional box of side 'a' each, the wavefunction of e^- are given by

$$\psi_{n_x, n_y, n_z} = \sqrt{\frac{8}{a^3}} \sin\frac{n_x \pi x}{a} \sin\frac{n_y \pi y}{a} \sin\frac{n_z \pi z}{a} \longrightarrow ①$$

and the energies are $E_{n_x, n_y, n_z} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right] \longrightarrow ②$

- As the maximum spacing between the energy levels that are consecutive is of the order of 10^{-6} eV. Hence, the distribution of energy is almost continuous (or) "quasi-continuous."

* Since e^- obey Pauli's exclusion principle i.e., only two e^- are allowed for a given orbital state, the probability of an e^- to occupy a given state can be represented as

$$\boxed{F(E) = \frac{1}{\left(1 + e^{\frac{E-E_F}{KT}}\right)}}$$

Here F(E) is the Fermi function, E is the energy of the level and E_F is the Fermi level which is Constant for a given system.

At absolute zero $F(E) = 0$ for $E > E_F$

$F(E) = 1$ for $E < E_F$

* at $E = E_F$, the Fermi function becomes ½. This signifies that it is the energy level for which the probability of occupation is ½.

DENSITY OF ENERGY STATES AND CARRIER DENSITY

Now we have to calculate the carrier concentration, i.e, the number of electrons per unit volume in a given energy range of interest. this is given by summing the product of the density of states $Z(E)$ and the occupancy probability $F(E)$ that is,

$$n_c = \int_{\text{Energy band}} Z(E)\,F(E)\,.dE \quad \text{— ①}$$

n_c = Carrier Concentration.

We know that the number of energy states with a particular value of E depends on how many combinations of quantum numbers result in the same value of n (degeneracy).

The number of energy states having energy values between E and E+dE (i.e, in the energy interval dE) can be shown to be

$$Z(E)\,dE = \frac{\pi}{2} n^2 dn \quad \text{— ②}$$

$$\because E = \frac{n^2h^2}{8mL^2}$$

$$\Rightarrow n^2 = \frac{8mL^2E}{h^2} \quad \text{— ③}$$

$$n = \left(\frac{8mL^2E}{h^2}\right)^{1/2} \quad — ④$$

Differentiating n^2 in eq ③.

$$2n\,dn = \frac{8mL^2}{h^2}\,dE$$

$$dn = \left(\frac{1}{2n}\right)\frac{8mL^2}{h^2}\,dE$$

$$= \left(\frac{8mL^2}{h^2}\right)\left(\frac{1}{2}\right)\left(\frac{h^2}{8mL^2}\right)^{1/2}\frac{dE}{E^{1/2}}$$

$$= \left(\frac{1}{2}\right)\left(\frac{8mL^2}{h^2}\right)^{1/2}\frac{dE}{E^{1/2}} \quad — ⑤$$

Substituting values of 'n^2' and 'dn' from eq ③ & eq ⑤ in eq ②.

$$Z(E)\,dE = \frac{\pi}{2} \times \frac{8mL^2E}{h^2} \times \frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{1/2}\frac{dE}{E^{1/2}}$$

$$Z(E)\,dE = \frac{\pi}{4} \times \left(\frac{8mL^2}{h^2}\right)^{3/2} \times E^{1/2}\,dE$$

According to pauli's Exclusion principle, two electrons of opposite ~~sign~~ spin can occupy each state, and hence the number of energy states available for electron occupancy is given by,

$$Z(E)\,dE = 2 \times \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{1/2}\,dE$$

$$= \frac{\pi}{2} \left(\frac{2^2 \times 2 m L^2}{h^2} \right)^{3/2} E^{1/2} \, dE$$

$$= \frac{\pi}{2} \times 8 \times \frac{L^3}{h^3} (2m)^{3/2} E^{1/2} \, dE$$

$$= \frac{4\pi}{h^3} L^3 (2m)^{3/2} E^{1/2} \, dE$$

Density of energy states is given by number of energy states per unit Volume.

i.e, Density of energy states, $Z(E)\,dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} \, dE$ — ⑥

Density of Charge Carriers,

Substituting the Value of $Z(E)$ and $F(E)$ in eq ①,

$$n_c = \int Z(E) \, F(E) \, dE .$$

$$= \frac{4\pi}{h^3} (2m)^{3/2} \int E^{1/2} \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)} \, dE \quad — ⑦$$

This is the expression for density of charge carriers at any temperature.

Equation ⑦ can be used to calculate the carrier concentration in metals and semiconductors.

BLOCH THEOREM FOR PARTICLE IN A PERIODIC POTENTIAL

→ If a electron moves through these ions, it experiences varying potentials.

→ The potential of electron at positive ion site is zero and is maximum in between two ions.

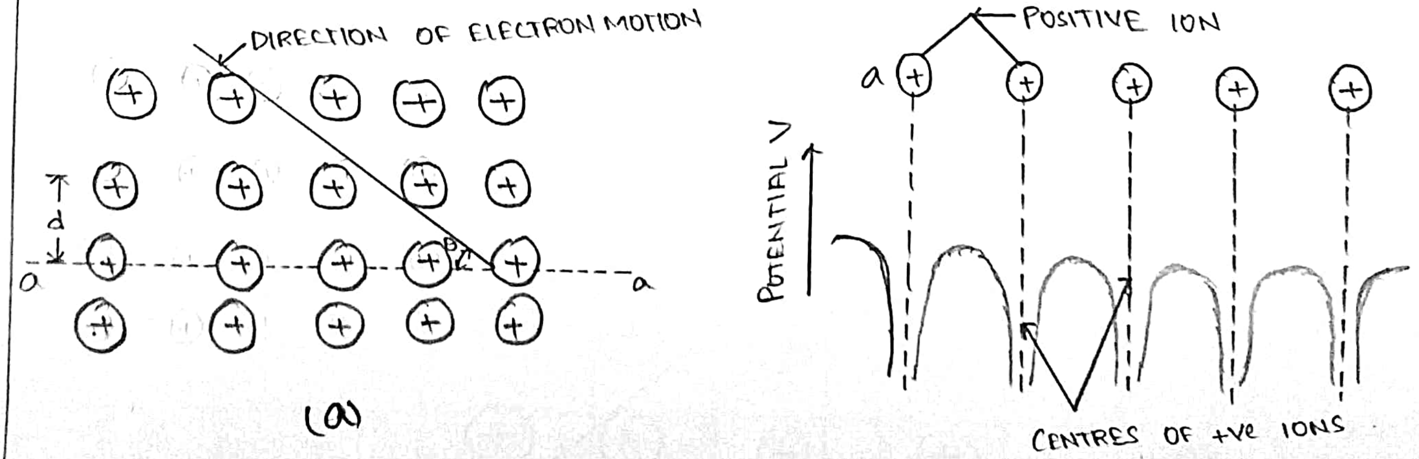
→ The potential experience by an electron varies periodically with the same period as the lattice.

→ The potential is negative because of attractive force between electron and positive ions.

→ Along x-direction in the crystal the potential function V_x has periodicity is $V_x = V(x+a)$. The energies of electrons can be known by solving schrödinger's wave equation in such a lattice.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V(x))\psi = 0.$$

→ Bloch showed a type of solution is where $u_K(x) = u_K(x+a)$.



(a)

(b)

PERIODIC POTENTIAL INSIDE A CRYSTAL

KRONIG – PENNEY MODEL :-

→ Sinusoidal potential shown in fig.b. It is not easy to solve Schrodinger's equation with these potentials.
So, Kronig and Penney approximated these potentials inside the crystal to the shape of rectangular steps shown in fig.c.

→ Schrodinger's equation solutions are possible only for

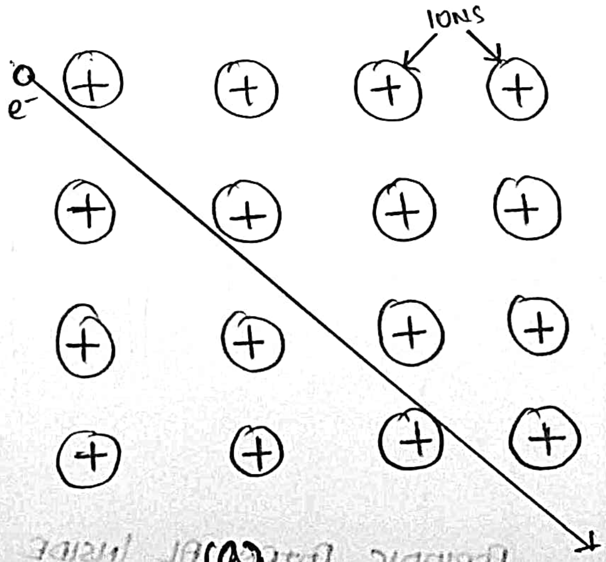
$$\cos Ka = \frac{P \sin \alpha a}{\alpha a} + \cos \alpha a .$$

→ Where, $P = \frac{4\pi^2 ma}{h^2} V_0 \omega$.

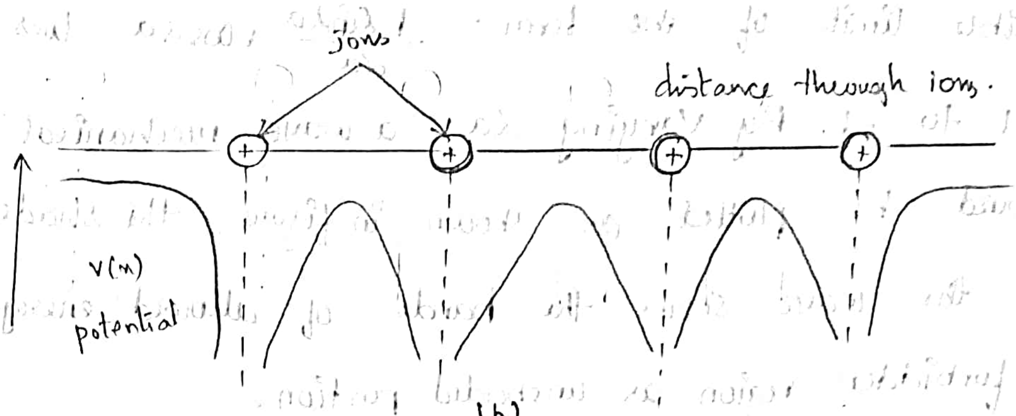
→ P is reffered to as the scattering power of the potential barrier and $V_0 \omega$ is called barrier strength.

→ where, $\alpha = \frac{2\pi}{h} \sqrt{2mE}$

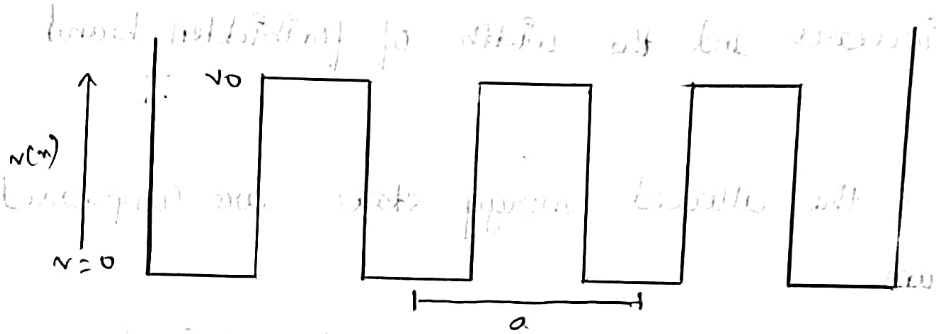
→ The $\cos Ka$ imposes limitations is +1 and −1. Hence, Certain range of value of α are allowed.



(a)



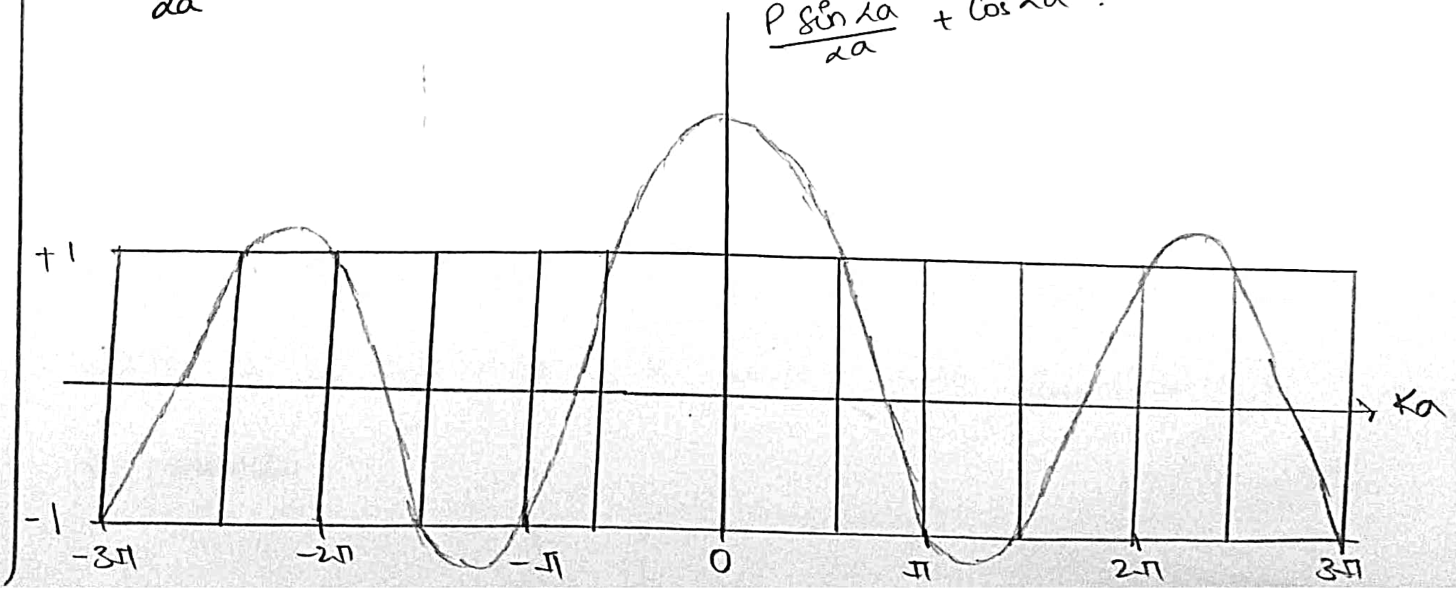
(b)



(c)

→ This means that energy E is restricted to lie within certain ranges which form the allowed energy band (or) zones it is called allowed band and some energies are not allowed it is called enegrgy gap.

→ This concept is understood by plotting a graph between the equation $\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a$ and Ka.

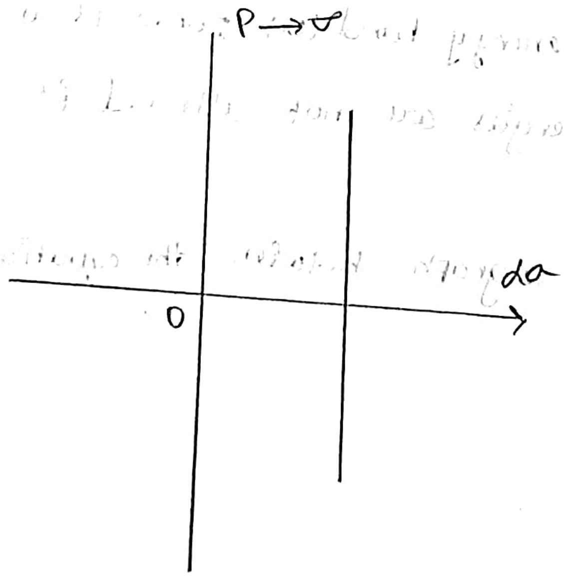


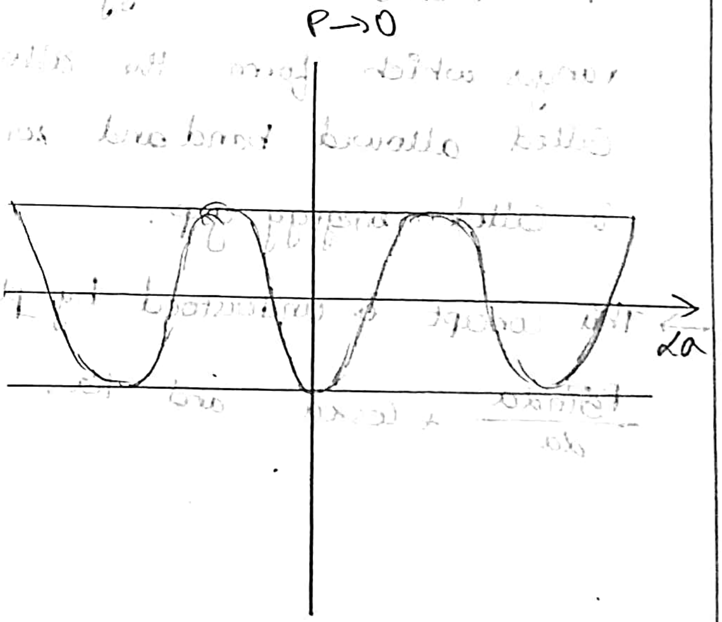
→ The permissible limit of the term = $\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a$ lies between +1 to -1. By varying αa, a wave mechanical nature could be plotted as shown in figure, the shaded portion of the wave shows the bands of allowed energy with the forbidden region as unshaded portion.

→ With increase of αa, band width of the allowed energy bands also increases and the width of foribidden band diminishes.

→ When $P = \infty$, the allowed energy states are compressed to a line spectrum.

→ When $P = 0$, the energy band is broadened and it is quasi continuous.





→ ~~The perm~~

BRILLOUIN ZONE OR E-K DIAGRAM

→ The Brillouin zone is represent of permissive value of k of the electron in one, two, and three dimensions.

→ Since Cos ka is a periodic function it will have same value whether it is ka is positive (or) negative and whether it is increased by integer multiple of 2π.

→ The total energy E of the electron is an even periodic function K with a period $\frac{2\pi}{\lambda}$.

→ Parabola representing the energy E of a free electron is compared with a periodic field.

The discontinuities in the parabola occur at values of K is

$K = \frac{n\pi}{a}$, where $n = \pm1, \pm2, \pm3 \ldots$

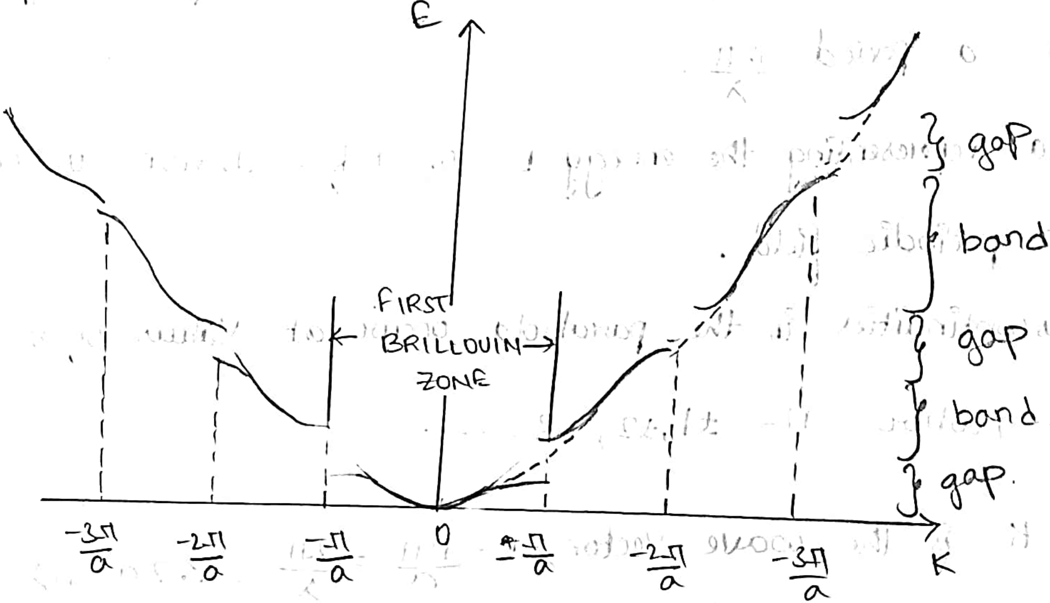
→ Since K is the wave vector, $K = \frac{n\pi}{a} = \frac{2\pi}{\lambda}$, $\therefore 2a = n\lambda$.
this is form of bragg's differaction law. Since, $\sin 90° = 1$.
So that electrons travelling through the periodic array of atoms in a crystal get diffracted.

→ Under this condition yields two standing waves, it shows that two electron position of energy are same value of K. This gives E-K curve with discontinuity in the parabola.

→ The dotted Curve shows the free electron parabola.

From this graph, the electron allowed energy Values in $K = -\frac{\pi}{a}$ to $+\frac{\pi}{a}$, this zone is Called first Brillouin zone.

$K = -\frac{\pi}{a}$ to $-\frac{2\pi}{a}$ and $+\frac{\pi}{a}$ to $+\frac{2\pi}{a}$ next allowed energy, Second Brillouin zone.... Similarly other higher order Brillouin zones Can be defined.



CONDUCTORS, SEMI CONDUCTORS AND INSULATORS

CONDUCTOR	SEMI CONDUCTOR	INSULATOR
→ It easily conducts the electric current.	→ It conducts the electric current less than conductor & greater than insulator.	→ Doesn't conduct any electric current.
→ Conductor formed using metallic bond.	→ formed due to covalent bonds.	→ formed due to ionic bonding.
→ Valence and conduction bands are overlapped.	→ bands are seperated by forbidden energy gap <3eV.	→ bands are seperated by forbidden energy gap >3eV.
→ Resistance is very small.	→ Resistance is high	→ Resistance is very high.
→ It has positive temperature coefficient	→ It has negative temperature coefficient	→ It has negative temperature coefficient
→ It has valence electrons in outermost orbit	→ It has 4 valence electrons in outermost orbit.	→ It has 8 valence electrons in outermost orbit.
Eg:- copper, aluminium.	Eg:- silicon.	Eg:- Mica.

EFFECTIVE MASS OF AN ELECTRON (m^*)

When an electron in a periodic potential accelerated by an Electric field (or) a magnetic field, then the mass of the electron varies with velocity. this means, the mass of the electron is function of velocity and is termed as effective mass of the electron.

Force acting on electron, $F = m^* a$.

$$a = \frac{F}{m^*} \qquad \text{—— } ①$$

According to quantum theory, an electron moving with velocity 'v' can be treated as wave packet moving with group velocity (V_g).

$$V_g = \frac{d\omega}{dK} \qquad \left[\because \begin{array}{l} \omega = \text{angular Velocity} \\ K = \text{wave vector} \end{array}\right]$$

$$V_g = \frac{d}{dK}(2\pi\nu)$$

$$V_g = 2\pi \frac{d}{dK}\left[\frac{E}{h}\right]$$

$$V_g = \frac{2\pi}{h}\frac{dE}{dK} \qquad \text{—— } ②$$

Differentiate eq ② with respect to time.

Acceleration,

$$a = \frac{d(V_g)}{dt} = \frac{d}{dt}\left[\frac{2\pi}{h}\frac{dE}{dK}\right]$$

$$a = \frac{2\pi}{h}\frac{d}{dt}\left[\frac{dE}{dK}\right] \times \frac{dK}{dK}$$

$$a = \frac{2\pi}{h}\frac{dK}{dt}\frac{d^2E}{dK^2} \quad \text{— ③}$$

We know that,

$$P = \hbar K$$

$$\frac{dP}{dt} = \hbar\frac{dK}{dt}$$

$$F = \hbar\frac{dK}{dt}$$

$$\frac{dK}{dt} = \frac{F}{\hbar} \quad \text{— ④}$$

Substituting eq ④ in eq ③,

$$a = \frac{2\pi}{h} \times \frac{F}{\hbar}\frac{d^2E}{dK^2}$$

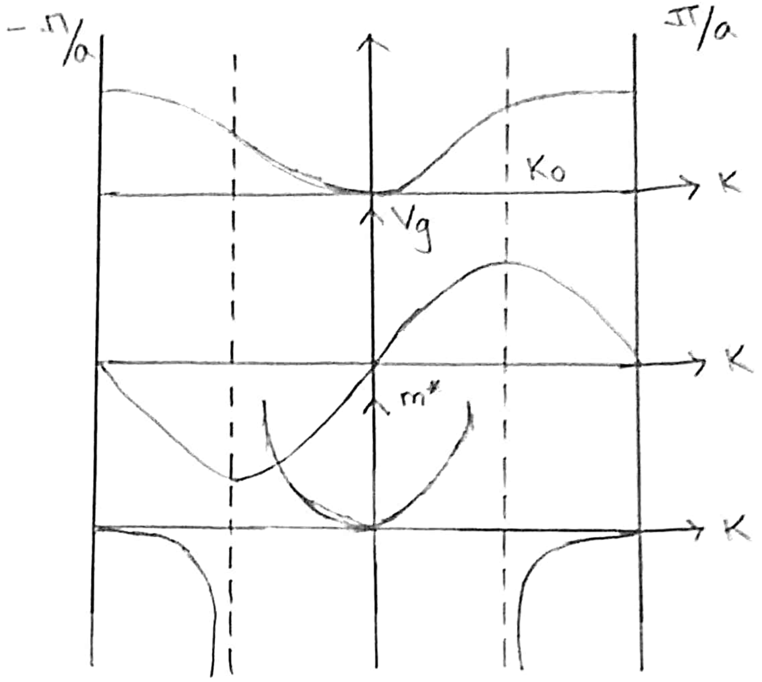
$$a = \frac{F}{\hbar^2}\frac{d^2E}{dK^2} \quad \text{— ⑤}$$

Comparing eq ⑤ with eq ①

$$\frac{F}{m^*} = \frac{F}{\hbar^2}\left[\frac{d^2E}{dK^2}\right]$$

$$m^* = \hbar^2 \Big/ \left[\frac{d^2E}{dK^2}\right] \quad \text{— ⑥}$$

SIGNIFICANCE :-



i) Upto inflection point, K_o,

Velocity increases (↑) ⇒ acceleration = positive (+ve),

$$\frac{d^2E}{dk^2} = \text{positive (+ve)} \quad , \quad m^* = \text{positive (+ve)}.$$

ii) At inflection point, K_o,

$$\text{Velocity} = \text{Constant} \Rightarrow \text{acceleration} = 0 \text{ ., } \frac{d^2E}{dk^2} = 0$$

$$m^* = \text{infinity } (\infty).$$

iii) Beyond inflection point, K_o.

$$\text{Velocity} = \text{decreases} \Rightarrow \text{acceleration} = \text{Negative (-ve)} \text{ , } \frac{d^2E}{dk^2} = \text{Negative (-ve)}$$

$$m^* = \text{negative (-ve)}$$

Negative (-ve) mass indicates, existence of holes in metals.

* Effective Mass of an Electron:

When an electron in a periodic potential accelerated by an electric field or a magnetic field. Then the mass of the electron varies with velocity. This means the mass of the electron is function of velocity and is termed as "effective mass of the electron (m^*)".

Force acting on the electron $F = m^* a$

$$a = \frac{F}{m^*} \longrightarrow ①$$

According to Quantum theory, an electron moving with velocity 'v' can be treated as wave packet moving with group velocity

$$V_g = \frac{d\omega}{dk} \quad \left[\begin{array}{l} \omega = \text{angular velocity} \\ k = \text{Wave vector} \end{array}\right]$$

$$V_g = \frac{d}{dk}(2\pi\nu) \quad [\because \omega = 2\pi\nu].$$

$$= 2\pi \frac{d\nu}{dk}$$

$$= 2\pi \frac{d}{dk}\left[\frac{E}{h}\right] \quad \left[\because E = h\nu \Rightarrow \nu = \frac{E}{h}\right]$$

$$= \frac{2\pi}{h} \cdot \frac{dE}{dk}$$

$$= \frac{1}{\hbar}\frac{dE}{dk} \quad [\hbar = h/2\pi] \longrightarrow ②$$

Acceleration $a = \frac{dV_g}{dt}$

Differentiate Eq. ② w.r.to 't'

$$a = \frac{dV_g}{dt} = \frac{d}{dt}\left[\frac{1}{\hbar}\frac{dE}{dk}\right]$$

$$a = \frac{1}{\hbar}\frac{d}{dt}\left(\frac{dE}{dk}\right) \times \frac{dk}{dk}$$

$$a = \frac{1}{\hbar}\frac{dk}{dt}\frac{d^2E}{dk^2} \longrightarrow ③$$

According to Moderm theory, we know $p = \hbar k \longrightarrow ④$

Differentiate Eq ④

$$\frac{dp}{dt} = \hbar\frac{dk}{dt}$$

$$F = \hbar\frac{dk}{dt}$$

$$\frac{dk}{dt} = F/\hbar \longrightarrow ⑤$$

Substitute Eq ⑤ in Eq. ③

$$a = \frac{1}{\hbar}\frac{F}{\hbar}\frac{d^2E}{dk^2}$$

$$\boxed{a = \frac{F}{\hbar^2}\frac{d^2E}{dk^2}.} \longrightarrow ⑥$$

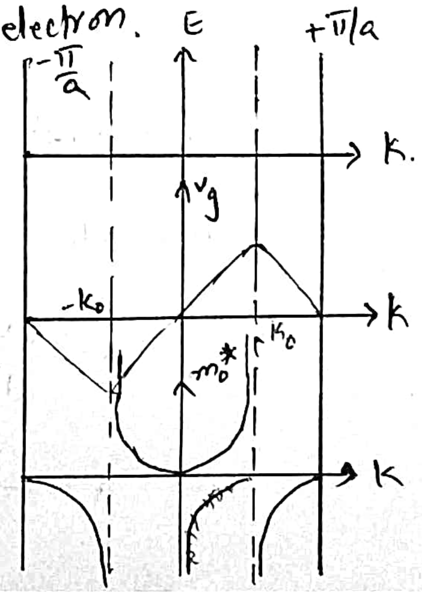
comparing Eq ① and Eq ⑥

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

$$m^* = \hbar^2\left[\frac{d^2E}{dk^2}\right]^{-1}$$

$$m^* = \left[\hbar^2 \Big/ \left[\frac{d^2E}{dk^2}\right]\right] \longrightarrow ⑦$$

The above Eq. gives effective mass of electron.



Up to Inflection point k_0

$V\uparrow \Rightarrow a = +ve \Rightarrow \frac{d^2E}{dk^2} = +ve$

$\boxed{m_e^* = +ve}$

At Inflection point k_0

$v = const. \Rightarrow a = 0 \Rightarrow \frac{d^2E}{dk^2} = 0 \Rightarrow m_e^* = \infty$

Beyond inflection point k_0

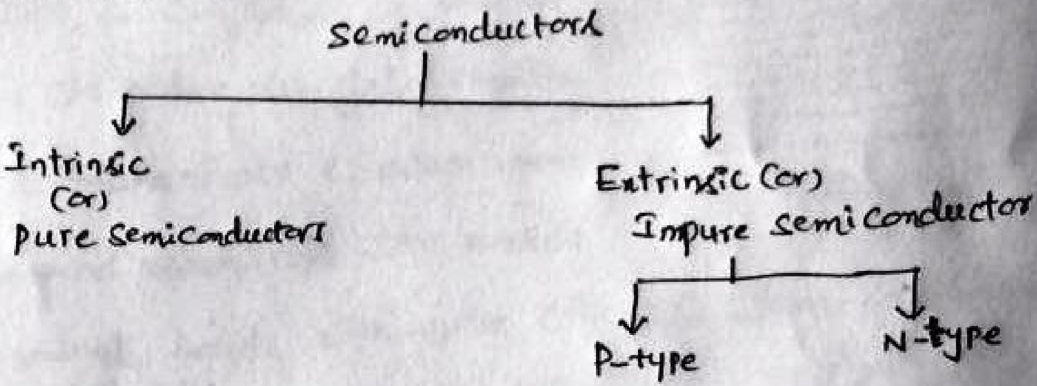
$V\downarrow \Rightarrow a = -ve \Rightarrow d^2E/dk^2 = -ve \Rightarrow m_e^* = -ve.$

-ve mass indicates Existance of holes. in metals.

SEMICONDUCTORS

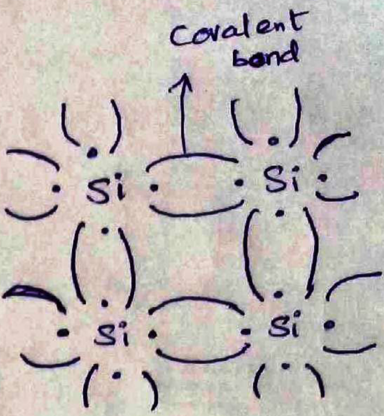
Introduction:- It has been observed that certain substances like Germanium (Ge), Silicon (Si) etc., have resistivity (10^{-4} to $0.5\ \Omega$-m) between good conductors like copper having resistivity $1.7\times10^{-8}\ \Omega$-m and Insulators like glass having resistivity $9\times10^{11}\ \Omega$-m. These substances are known as semiconductors. "Thus a substance which has resistivity in between conductors & insulators is known as semiconductors"

Semiconductors are broadly classified as under



* Semiconductors have the following properties
  - i) They have resistivity lessthan Insulators & morethan conductors.
  - ii) The resistivity of semiconductor decreases with the increase in temperature and vice versa. i.e. They have negative Temperature coefficient of resistance (-ve TCR).
  - iii) When suitable mettalic impurity like arsenic, gellium etc is added to a semiconductor, its current conducting properties change appreciably. This is the most imp. property.

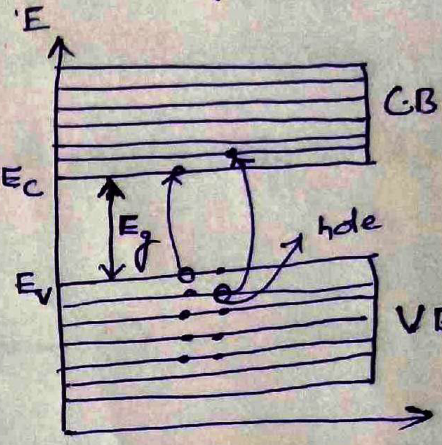
i) Intrinsic Semiconductor :-



fig(1): Si crystal

A semiconductor which doesnot have any kind of impurities, behaves as an insulator at $T = 0K$ and behaves as a conductor at high temperature is known as Intrinsic Semiconductor.

Ex: Si, & Ge, etc.

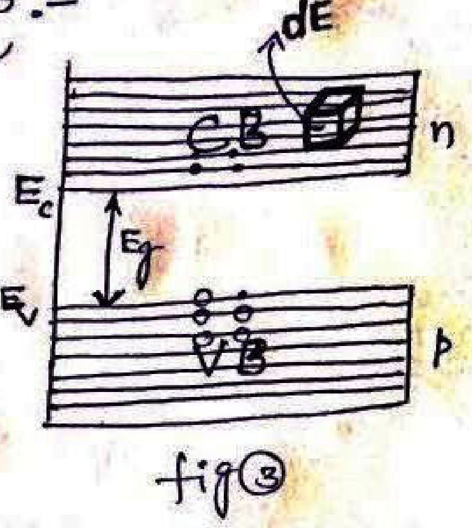


fig②

In order to get insight view of an intrinsic Semiconductor, let us Consider Si, Which has 4 Valency electrons. In order to gain stability it has to make four covalent bonds. In this regard each Si atom makes four covalent bonds with four other Si atoms as shown in fig①.

- * The $\bar{e}$s which are participating in the covalent bonds are known as Valence $\bar{e}$s.
- * If some energy is supplied then covalent bond break, $\bar{e}$s will come out and move freely, resulting in the formation of vacant sites in the covalent bonds. These are known as positive charge carriers named as 'holes' as shown in ②
- * The no. of free or conduction $\bar{e}$s will be equal to the no. of vacant sites or holes in the valance band $(n = p)$

Carrier Concentration (density) of e^-s in C.B :-



fig③

Let 'n' be the no. of e^-s in the Conduction band per unit volume of a homogeniously doped semiconductor crystal in equilibrium. Let us consider a small element in the energy range of E and E+dE. Let 'dn' is the no. of electrons in the range E and E+dE in Conduction band.

$$\therefore \; dn = Z(E) \cdot F(E)\, dE \quad \text{—} ①$$

Where, Z(E) is the no. of energy states per unit volume and F(E) is the Probability of electron in Conduction band in the range E and E+dE.

$$Z(E)\, dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2} E^{1/2}\, dE$$

Energy level in the CB is above the E_c $(E>E_c)$

$$\therefore \; Z(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2}\, dE \quad \text{—} ②$$

Fermi dirac distribution function, $F(E) = \dfrac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)}$

But, $E-E_F \gg KT \Rightarrow F(E) = \left[1+\exp\left(\frac{E-E_F}{KT}\right)\right]^{-1}$

$$\therefore \; F(E) = \exp\left[\frac{-(E-E_F)}{KT}\right]$$

$$\therefore \; F(E) = \exp\left(\frac{E_F-E}{KT}\right) \quad \text{—} ③$$

If the Conduction band extends from E_C to ∞, then the electron Concentration can be written as

From eq① $\Rightarrow n = \int_{E_c}^{\infty} dn = \int_{E_c}^{\infty} Z(E)dE\,F(E)$

From eq ② & ③ $\Rightarrow$

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2}\exp\left(\frac{E_F-E}{KT}\right)dE$$

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}\exp\left(\frac{E_F}{KT}\right).\int_{E_c}^{\infty}(E-E_c)^{1/2}\exp\left(\frac{-E}{KT}\right)dE$$

Put $E-E_c=x \Rightarrow dE=dx$, Limits $x=0$, & $x=\infty$

$\Rightarrow E=E_c+x$

$$\therefore n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}\exp\left(\frac{E_F}{KT}\right)\int_0^{\infty} x^{1/2}\exp\left(\frac{-(E_c+x)}{KT}\right)dx$$

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}\exp\left(\frac{E_F}{KT}\right).\exp\left(\frac{-E_c}{KT}\right).\int_0^{\infty} x^{1/2}\exp\left(\frac{-x}{KT}\right)dx$$

But from standard Integral, $\int_0^{\infty} x^{1/2}\exp\left(\frac{-x}{KT}\right)dx = (KT)^{3/2}\frac{\sqrt{\pi}}{2}$

$$\therefore n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}\exp\left(\frac{E_F-E_c}{KT}\right).(KT)^{3/2}\frac{\sqrt{\pi}}{2}$$

$$\boxed{n = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}\exp\left(\frac{E_F-E_c}{KT}\right)}$$

$$\boxed{n = N_c\exp\left(\frac{E_F-E_c}{KT}\right)}$$

where $N_c = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}$

This expression is called the carrier concentration of $\bar{e}$s in CB

Carrier Concentration of holes in VB :-

Let dp be the no. of holes (vacancies) in the energy Interval E and E+dE in the Valance band.

$$dp = Z(E)\,[1-F(E)]\,dE \quad \text{—— ①}$$

Where Z(E)dE is the density of states per unit volume in the energy interval E and E+dE and [1-F(E)] is the probability of existance of a hole. (since F(E) is the probability of e^- occupancy, 1-F(E) gives the probability of an e^- not occupying the state. i.e, presence of a hole).

$$1 - F(E) = 1 - \left[1 + \exp\left(\frac{E-E_F}{KT}\right)\right]^{-1} \qquad \left(\because x = e^{\frac{E-E_F}{KT}}\right)$$

Since $E-E_F >> KT$,
$$1-F(E) = 1-[1+x]^{-1}$$
$$= 1-\left(1-\frac{x}{1!}+x^2-x^3+x^4+ - - -\right)$$
$$= 1-(1-x) = x$$

$$\therefore 1-F(E) = \exp\left(\frac{E-E_F}{KT}\right) \quad \text{—— ②}$$

density of states in the interval E & E+dE in the Valence band is

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}E^{1/2}dE$$

Since E_V is the Energy of the Top of the Valence band,

$$\therefore Z(E)\,dE = \frac{4\pi}{h^2}(2m_h^*)^{3/2}(E_V-E)^{1/2}dE \quad \text{—— ③}$$

To calculate the no. of holes in the V.B., the eqn ① has to be integrated from $-\infty$ to E_V

The no. of holes in the V.B. per unit volume is given by.

$$p = \int_{-\infty}^{E_V} dp = \int_{-\infty}^{E_V} \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V-E)^{1/2}\exp\left(\frac{E-E_F}{KT}\right)dE$$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\exp\left(\frac{-E_F}{KT}\right)\int_{-\infty}^{E_V}(E_V-E)^{1/2}\cdot\exp\left(\frac{E}{KT}\right)dE$$

put $E_V - E = x \Rightarrow E = (E_V - x) \Rightarrow dE = -dx$

Limits $E = -\infty \Rightarrow x = \infty$
$\quad\;\; = E_V \Rightarrow x = 0$ }

$$\therefore p = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\exp\left(\frac{-E_F}{KT}\right)(-)\int_{\infty}^{0} x^{1/2}\exp\left(\frac{E_V - x}{KT}\right)dx$$

$$= \frac{4\pi}{h^3}(2m_h^*)^{3/2}\exp\left(\frac{-E_F}{KT}\right)\cdot\exp\left(\frac{E_V}{KT}\right)\cdot\int_{0}^{\infty} x^{1/2}\exp\left(\frac{-x}{KT}\right)dx$$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\exp\left(\frac{E_V-E_F}{KT}\right)\cdot(KT)^{3/2}\frac{\sqrt{\pi}}{2}$$ (using standard Integral)

Hence

$$\boxed{p = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{KT}\right)}$$

i.e., the no. of holes in the V.B. per unit volume is given by,

$$\boxed{p = N_V \exp\left(\frac{E_V-E_F}{KT}\right)} \qquad \because N_V = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2}$$

Intrinsic Carrier Concentration :-

In intrinsic semiconductors, $n = p$

Hence $n = p = n_i$ is called intrinsic carrier concentration.

Therefore $\boxed{n_i^2 = n \cdot p}$

$$n_i^2 = n\cdot p = 4\left(\frac{2\pi KT}{h^2}\right)^3 \left(m_e^* m_h^*\right)^{3/2} \exp\left(\frac{E_V - E_C}{KT}\right)$$

$$= 4\left(\frac{2\pi KT}{h^2}\right)^3 \left(m_e^* m_h^*\right)^{3/2} \exp\left(\frac{-E_g}{KT}\right)$$

Where $E_C - E_V = E_g$ is the forbidden energy gap,

Hence, $$\boxed{n_i = 2\left(\frac{2\pi KT}{h^2}\right)^{3/2} \left(m_e^* m_h^*\right)^{3/4} \exp\left(\frac{-E_g}{2KT}\right)}$$

This is the Expression for intrinsic carrier concentration.

Fermi Level :-

Since $n = p$ in intrinsic semiconductors,

$$2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) = 2\cdot\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_V - E_F}{KT}\right)$$

or. $$\left(m_e^*\right)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) = \left(m_h^*\right)^{3/2} \exp\left(\frac{E_V - E_F}{KT}\right)$$

$$\exp\left(\frac{2E_F}{KT}\right) = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} \cdot \exp\left(\frac{E_C + E_V}{KT}\right)$$

Taking logarithms on both sides,

$$\frac{2E_F}{KT} = \frac{3}{2}\log\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_C + E_V}{KT}$$

$$\therefore\ E_F = \frac{3KT}{4}\log\left(\frac{m_h^*}{m_e^*}\right) + \left(\frac{E_C + E_V}{2}\right)$$

If we assume that $m_e^* = m_h^* \Rightarrow \boxed{E_F = \frac{E_C + E_V}{2}}$

Thus fermi level is located half way B/w the VB & CB and its position is independent of Temperature.

ii) Extrinsic SemiConductors:-

An extrinsic semiconductor is an intrinsic semiconductor doped by a specific impurity (B, In, P, etc.) which is able to modify deeply its electrical properties, making it suitable for electronic or opto electronic applications.

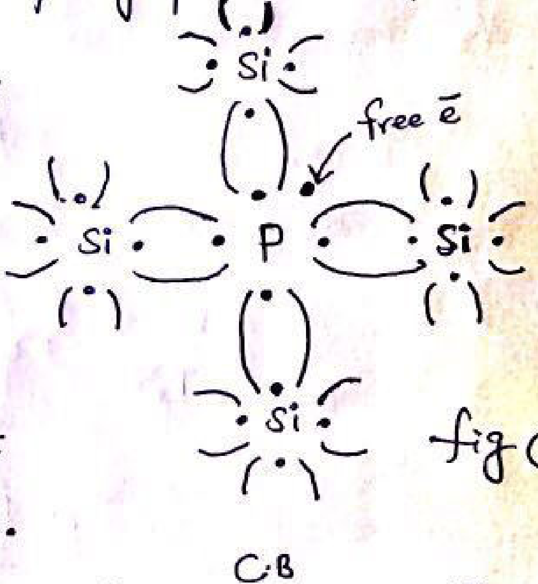
Extrinsic semiconductors are basically of two types:

b). p-type semiconductor

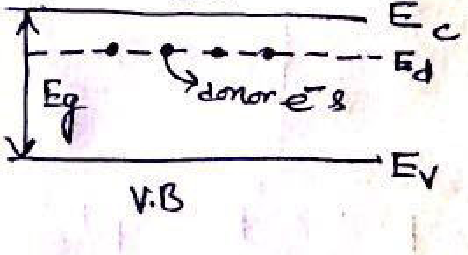
a) n-type semiconductor

a). n-type semiconductor:- when pentavalent impurity is added to an intrinsic semiconductor (Si or Ge), then it is said to be an n-type semi conductor. pentavalent (+5) impurities such as P, As, Sb, etc, are called donor Impurity

Let us consider, pentavalent impurity P is added to Si as shown in fig①. P atom has 5 Valence $\bar{e}$s and Si has 4 valence $\bar{e}$s. P atom has one excess valence e^- than Si. The 4 valence $\bar{e}$s of each P atom form 4 covalent bonds with 4 neighbouring Si atoms. The fifth valence e^- of P atom does not involve in the formation of covalent bonds. Hence it is free



fig①



to move and not attached to the parent atom. This shows that each P atom donates one free $\bar{e}$. Therefore, all the pentavalent impurities are called donors. The no. of donor electrons are depends on the amout of impurity (P) added to the Si.

In case of n-type semiconductor, the following points should be remembered:

i) In n-type semiconductor, the $\bar{e}$s are majority carriers while holes are minority carriers

ii). n-type semiconductor has excess $\bar{e}$s but it is electrically neutral. This is due to the fact that $\bar{e}$s are created by the addition of neutral V-valent impurity atoms to the S.C.

iii). In n-type S.C, In addition to the free $\bar{e}$s of impurity atoms, the density of free $\bar{e}$s is given by

$$\boxed{n = n_i + N_d}$$

Where n_i is the density of intrinsic S.C. $\bar{e}$s and N_d, the density of ionised donor atoms formed due to thermal aggitation.

$$N_d > n_i \quad \therefore \quad n \approx N_d$$

In a purely n-type crystal, the minority charge density is given by $\boxed{P_n = \frac{n_i^2}{n} = \frac{n_i^2}{N_d}}$

Fermi level in n-type S.C. and Variation with temperature :-

The energy level diagram of n-type S.C is shown in fig① N_d is the donor Concentration

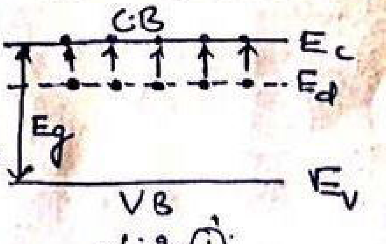


fig ①

i.e., the no. of donor atoms per unit volume of the material and E_d is the donor energy level. At very low temperatures all the donor levels are filled with electrons. With increase of temperature more & More donor atoms gets ionized & the density of $\bar{e}$s in the CB. increases. Density of electrons in the Conduction Band according to pure Si atoms (intrinsic).

$$n = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_c}{KT}\right) \quad ①$$

donor atom ($\bar{e}$) concentration in the energy level E_d is n_d

$$n_d = N_d[1-F(E_d)] \approx N_d \exp\left(\frac{E_d - E_F}{KT}\right)$$

At very low temperatures, $n = n_d$

$$2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_c}{KT}\right) = N_d \exp\left(\frac{E_d - E_F}{KT}\right)$$

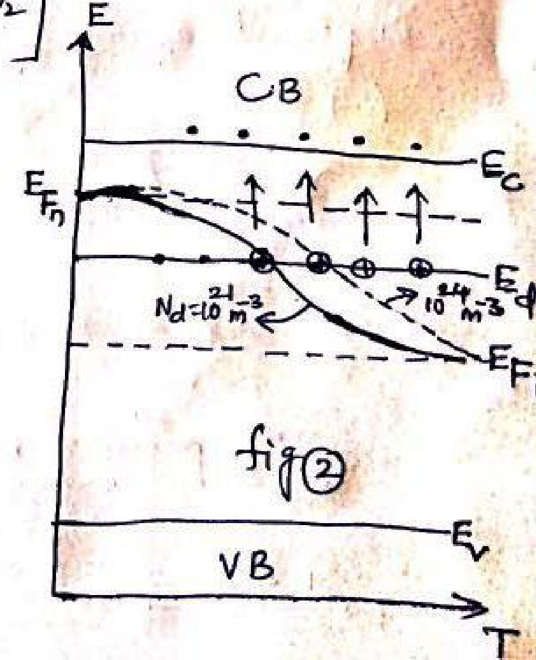
Taking logarithm on both sides, we get

$$\left(\frac{E_F - E_c}{KT}\right) - \left(\frac{E_d - E_F}{KT}\right) = \log N_d - \log 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}$$

$$2E_F - (E_d + E_c) = KT \log\left[\frac{N_d}{2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}}\right]$$

$$E_F = \frac{E_d + E_c}{2} + \frac{KT}{2} \log\left[\frac{N_d}{2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}}\right]$$

At T = 0K, $\boxed{E_{F_n} = \frac{E_d + E_c}{2}}$



fig②

With increase of temperature, more and more donor atoms are ionised and also starts the generation $\bar{e}$-hole pairs. The Fermi level moves gradually towards the intrinsic Fermi level E_{F_i} as shown in fig②

ⓑ p-Type Semiconductor :-

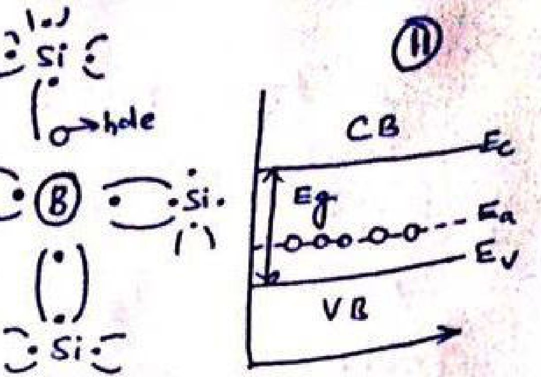


fig ①.

When a small amount of trivalent impurity is added to a pure (intrinsic) Semiconductor. Then it is said to be p-type Semiconductor.

Trivalent impurities (+3) such as B, Ga, In, etc.,. Let us consider the case when trivalent Boron (B) is added to pure Si crystal as shown in fig ①. As shown in fig ① each atom of B fits into Si crystal with only three covalent bonds. In the forth covalent bond, only Si atom contributes one valence e^- and there is a deficiency of one electron which is called a 'Hole'. The addition trivalent impurity atoms, produces large no. of holes are termed as "acceptors" because they accept e^-s from Si atoms. Since current carriers are +vely charged (holes), This type of semi-conductor is called p-type S.C.

i) In p-type S.C. materials, the majority carriers are 'holes' and minority carriers are e^-s

ii) In p-type S.C., there are holes due to acceptor impurity in addition to free e^-s & holes of intrinsic S.C.

iii) The density of holes in p-type-s.c. at Room temperature is

$$p = p_i + N_a$$

where N_a is the acceptor concentration, p_i - concentration of intrinsic holes.

In p-type - S.C., $N_a >> p_i$

$$\therefore \boxed{p \approx N_a}$$

free electron concentration (Minority)

$$\boxed{n_p = \frac{n_i^2}{p} = \frac{n_i^2}{N_a}}$$

Fermi level in p-type s.c & its variation with temperature & acceptor concentration :-

The energy level diagram of p-type semiconductor is shown in fig ①. Na is the acceptor concentration (The no. of acceptor atoms per unit volume of the material) and E_a is the acceptor energy level slightly above the V.B. edge E_v as shown in fig ①.

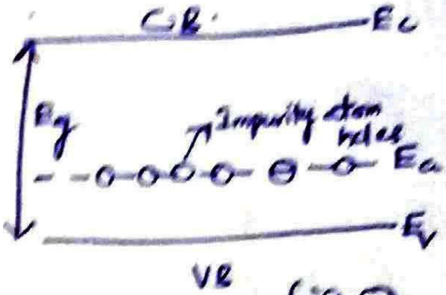


fig ①

Concentration of holes (intrinsic) in valence band is given by,

$$p = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_F}{KT}\right)$$

If some of the doped atoms get ionized by accepting the electrons, let Na be the acceptor concentration,

$$= N_a F(E_a) = N_a\left[1 + \exp\left(\frac{E_a - E_F}{KT}\right)\right]^{-1}$$

$$= N_a \exp\left(\frac{E_F - E_a}{KT}\right)$$

Since the concentration of holes in the V.B. is equal to the concentration of ionized acceptors,

$$2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_F}{KT}\right) = N_a \exp\left(\frac{E_F - E_a}{KT}\right)$$

$$\exp\left(\frac{E_v + E_a - 2E_F}{KT}\right) = \frac{N_a}{2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2}}$$

Taking logarithm,

$$E_{F_p} = \frac{E_v + E_a}{2} - \frac{KT}{2}\log\frac{N_a}{2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2}}$$

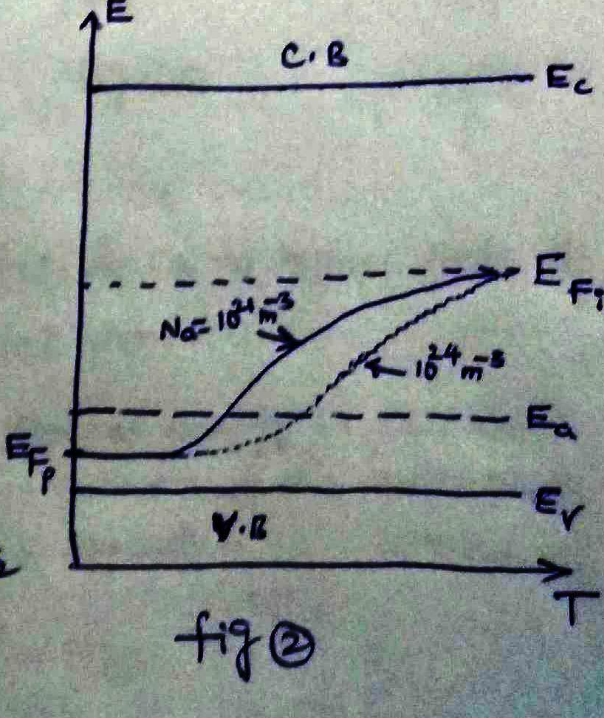


fig ②

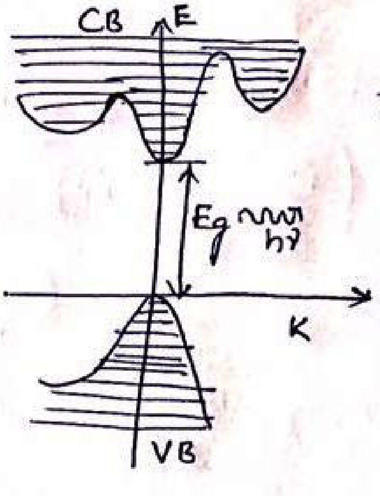
At T = 0 K, $\boxed{E_{F_p} = \frac{E_v + E_a}{2}}$

As the temperature is increased, more and more accptor atoms are ionized and results in generation of e^--hole pairs due to breaking of covalent bondsand the material tends to behave in intrinsic manner. The Fermi level of Extrinsic p-type S.C gradually moves towards the intrinsic fermi level E_{F_i} as shown in fig②.

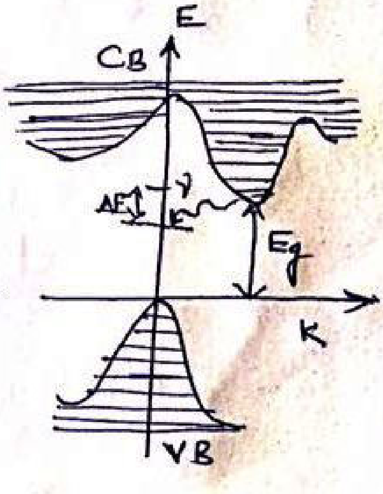
—— * ——

Direct and Indirect Band gap Semiconductors:-

1) **Direct Band gap:-** The minimum energy state in the CB, and the maximum energy state in the VB. are characterized by same K-vector, it forms direct band gap Semiconductor as shown in fig ⓐ(1a).



fig①ⓐ



fig①ⓑ

* An e^- can shift from the lowest state of CB to higher state of V.B without any change in crystal momentum.

* When an e^- and hole combine, the excess energy is released in the form of photon & this process is called radiative recombination,

* It obeys conservation of energy & Momentum

Ex:- GaAs, InP etc.,

ii) Indirect bandgap:- Indirect Band gap is a band gap in which the minimum of C.B and maximum of the V.B have different K-vector. The Semiconductors in which the Recombination of charge carriers through phonon are called Indirect Band gap Semiconductors (fig (1b).).

Semiconductors that have an indirect band gap are inefficient at emitted light. This is because any ē's present in the C.B. quickly settle into the Energy minimum of that band. Electrons in this minimum require some source of momentum allowing them to overcome the offset and fall into the V.B. Here, there is emission of photon along with phonon.

$$\boxed{E_g = h\nu + E_{phonon}}$$

* life-time of the charge carriers is more compared to Direct band gap S.C.

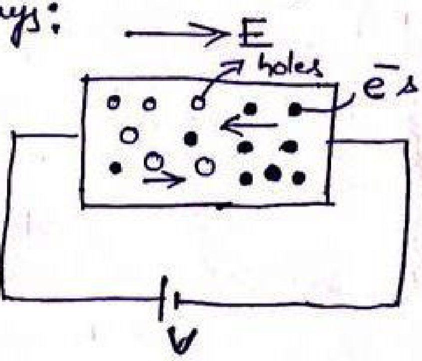
* These are mostly from the elemental S.C's

Ex:- Si, Ge. etc.,

* * *

DRIFT & DIFFUSION :- The net flow of current through a S.C material in two ways:

i) Drift Current and Conductivity:-



The charge carriers are forced to move in a particular direction in the presence of external electric field E, constitute the Electric Current. This phenomenon is known as the 'Drift'.

Let us Consider an n-type S.C with a large no. of free $\bar{e}$s. An Electric field E is applied in one direction produces a force on the $\bar{e}$s in the opposite direction, because of the $\bar{e}$s -ve charge. The e^-s acquire a drift velocity V_{dn}, which can be written as

$$V_{dn} = -\mu_n E \quad \text{—} ①$$

Where μ_n is a constant called the electron mobility.

The electron drift produces a drift current density given by

$$J_n = -nev_{dn} = -ne(-\mu_n E)$$

$$\boxed{J_n = ne\mu_n E} \quad \text{—} ②$$

Where n is the electron concentration

$$\text{Conductivity}, \ J = \sigma E \Rightarrow \sigma_n = \frac{J_n}{E} = ne\mu_n$$

$$\boxed{\sigma_n = ne\mu_n} \quad \text{—} ③$$

In the case of p-type S.C,

Drift Current density due to holes is

$$\boxed{J_p = pe\mu_p E} \ \& \ \boxed{\sigma_p = pe\mu_p}$$

Then the total drift Current density

$$\boxed{J_{drift} = J_n(\text{drift}) + J_p(\text{drift})}$$

$$J = ne\mu_n E + pe\mu_p E$$

$$\boxed{J = eE(n\mu_n + p\mu_p)} \ \& \ \boxed{\sigma = \frac{J}{E} = en\mu_n + pe\mu_p}$$

For an Intrinsic Semiconductor, $n = p = n_i$

$$\therefore \boxed{J_{drift} = n_i e E(\mu_n + \mu_p)}$$

$$\boxed{\sigma_i (drift) = n_i e (\mu_n + \mu_p)}$$

* * *

ii) Diffusion Current & Conductivity :-

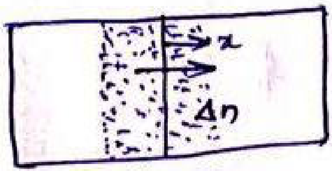


fig ①

Due to non-uniform carrier concentration in a semiconducter, the charge carriers moves from a region of higher concentration to a region of lower concentration. This process is known as diffusion of charge carriers.

Let us consider the n-type semiconductor, in which Δn be the excess electron concentration. Then according to Fick's law, the rate of diffusion of electrons,

$$\propto \frac{-\partial(\Delta n)}{\partial x}$$

$$= -D_n \frac{\partial(\Delta n)}{\partial x} \quad \text{———} ①$$

Where D_n is the Diffusion Coefficient of e^-s, the diffusion current density due to e^-s is given by J_n

$$J_n = -e\left[-D_n \frac{\partial(\Delta n)}{\partial x}\right]$$

$$J_n = e D_n \frac{\partial(\Delta n)}{\partial x} \quad \text{———} ②$$

The diffusion Current density due to holes

$$J_p(\text{diffusion}) = +e\left[-D_p\frac{\partial}{\partial x}(\Delta P)\right] = -eD_p\frac{\partial(\Delta P)}{\partial x} \quad \text{——③}$$

The Total current density due to e^-s is the sum of the current densities due to drift & diffusion of e^-s.

$$J_n = J_n(\text{drift}) + J_n(\text{diffusion})$$

$$\boxed{J_n = ne\mu_n E + eD_n\frac{\partial}{\partial x}(\Delta n)}$$

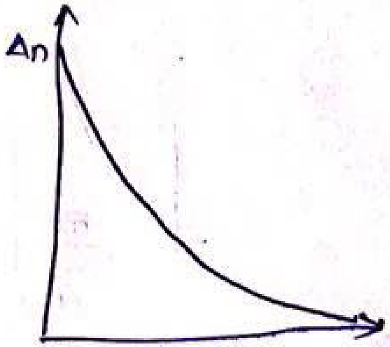


fig ②

IIly

$$J_p = pe\mu_p E + \left(-eD_p\frac{\partial}{\partial x}(\Delta P)\right)$$

$$\Rightarrow \boxed{J_p = Pe\mu_p E - eD_p\frac{\partial}{\partial x}(\Delta P)}$$

The variation of excess electron density Δn w.r.t x is shown in fig ②..

——— × ———

EINSTEIN'S Relations:-

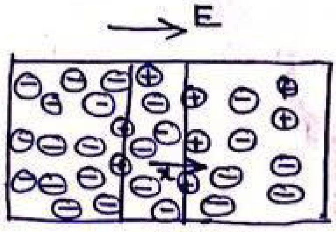


fig ①

The relation between mobility μ and Diffusion coefficient D of charge carriers in a semiconductor is known as Einstein's relation. At equilibrium with no applied field, the charge distribution is uniform, there is no net current flow. Any disturbance in equilibrium state leads to diffusion of charge carriers resulting in a diffusion current which creates an Internal $\vec{E}$. This field causes the drifting of charge carriers resulting in a drift current. At equilibrium condition, the drift current & diffusion current balance each other.

$$\therefore \quad \Delta n e\mu_n E = eD_n\frac{\partial(\Delta n)}{\partial x} \quad \text{——①}$$

The force on excess electrons restoring equilibrium is the Product of excess charge and E, i.e $F = (\Delta n) eE$

from eq ① ⇒ $F = \left[\frac{eD_n}{\mu_n}\right] \frac{\partial(\Delta n)}{\partial x}$ —— ②

From Kinetic theory of gases, the force on gas molecules is given by, $F = K_BT \frac{\partial(\Delta n)}{\partial x}$ —— ③

Comparing eqn ② & ③, we get,

$$K_BT = \frac{eD_n}{\mu_n} \Rightarrow \boxed{\frac{D_n}{\mu_n} = \frac{K_BT}{e}} \text{ —— ④}$$

Similarly for holes, we get,

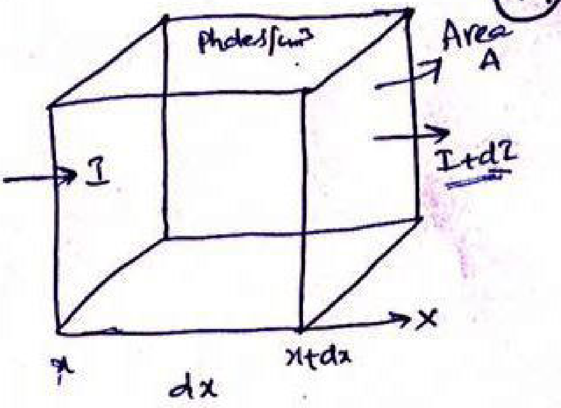
$$K_BT = \frac{eD_p}{\mu_p} \Rightarrow \boxed{\frac{D_p}{\mu_p} = \frac{K_BT}{e}} \text{ —— ⑤}$$

from eqn ④ & ⑤, we get $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} \Rightarrow \boxed{\frac{D_n}{D_p} = \frac{\mu_n}{\mu_p}}$ —— ⑥.

∴ Equation ⑥. is Known as Einstein's Relation.

*

CONTINUITY EQUATION :- The equation governing the behaviour of charge carriers in a semiconductor is known as "equation of continuity". This equation gives the condition of dynamic equilibrium for the density of charge carriers in any elementary volume of a semiconductor.



fig①.

Let us consider the infinitesimal volume element of area A & length dx as shown in fig ①. Let 'p' be the Avg hole concentration within this volume. If τ_p is the mean life time of the holes,

The holes lost per sec per unit volume by recombination is $= \frac{p}{\tau_p}$

The rate of loss of charge with in the volume is

$$= e\,A\,dx\,\frac{p}{\tau_p} \quad \text{—— ①}$$

If 'g' is the thermal generation coefficient of hole-$\bar{e}$s pairs per unit volume,

The rate of increase of charge with in the volume due to generation is $= e\,A\,dx\,g$ —— ②

If I is the current entering the volume at x and I+dI is the current leaving the volume at x+dx, Then

the decrease of charge per sec from the volume is $= dI$ —— ③

∴ Increase in the hole concentration per sec with in the volume is

$$e\,A\,dx\,\frac{dp}{dt} \quad \text{—— ④}$$

According to conservation of charge, charge can be neither be created nor destroyed.

Increase = generation − loss

from ①,②③ & ④, we get,

$$eAdx\frac{dp}{dt} = eAdx\,g - eAdx\frac{p}{\tau_p} - dI \quad ⑤$$

$$\frac{dp}{dt} = g - \frac{p}{\tau_p} - \frac{dI}{eAdx} \quad ⑥$$

When no External field is applied, under thermal equilibrium condition, the hole density attains constant value i.e. p_0,

$$\therefore dI = 0, \quad \frac{dp}{dt} = 0$$

From, eq ⑥, we get, $g = \frac{p_0}{\tau_p}$ — ⑦

from eq, ⑥, we get, $\frac{dp}{dt} = \frac{p_0 - p}{\tau_p} - \frac{dI}{eAdx}$ — ⑧

$$\frac{dp}{dt} = \frac{p_0 - p}{\tau_p} - \frac{dJ}{e\,dx} \quad \left(\because J = \frac{I}{A} \Rightarrow dI = A\cdot dJ\right)$$

$$\frac{dp}{dt} = \frac{p_0 - p}{\tau_p} - \frac{1}{e}\frac{dJ}{dx} \quad ⑧$$

The Resultant current density of holes due to drift & diffusion is given by $J_p = J_p(\text{drift}) + J_p(\text{diffusion})$

$$\therefore J_p = pe\mu_p E - eD_p\frac{dp}{dx} \quad ⑨$$

Substitute eqn ⑨ in eqn(8), we get

$$\frac{dp}{dt} = \frac{p_0 - p}{\tau_p} - \frac{1}{e}\cdot e\frac{d}{dx}\left[p\mu_p E - D_p\frac{dp}{dx}\right]$$

$$\boxed{\therefore\ \frac{dP}{dt} = \frac{P_0 - P}{\tau_p} - \mu_p E \frac{dP}{dx} + D_p \frac{d^2 p}{dx^2}}$$

The above eqn is the Continuity eqn for holes. Similarly, the Continuity equation for electrons is

$$\boxed{\frac{dn}{dt} = \frac{n_0 - n}{\tau_n} - \mu_n E \frac{dn}{dx} - D_n \frac{d^2 n}{dx^2}}$$

$\times$

HALL EFFECT:- When a magnetic field is applied ⊥r to a current carrying conductor or semiconductor, a voltage is developed across the specimen in a direction ⊥r to both the current and the magnetic field. This phenomenon is called the "HALL EFFECT" and the voltage so developed is called the "HALL VOLTAGE".

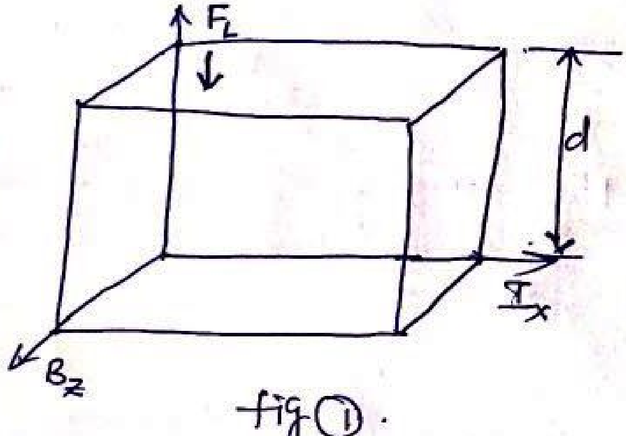


fig ①.

Explanation:- Let us consider a uniform thick semiconductor strip of thickness 'd' is placed with its length parallel to x-axis.

Let a current 'i_x' is passed in the strip along x-axis and a magnetic field 'B_z' is established along z-axis. Due to the magnetic field, the charge carriers experience a force F_B ⊥r to x-z plane i.e. along y-axis. The direction of this force is given by Fleming Left hand rule. If the charge carriers are ē's then they will experience a force in the -ve direction of Y. Hence, the Bottom surface will be -vely

Charged while the top surface will be charged +vely. In a p-type S.C, the Bottom surface is accumulated with holes and it becomes more +ve compared to the top surface. Thus, a transverse potential difference is created. This is called 'Hall voltage', V_H.

The Resultant force along y-direction is

$$|F_L| = |F_E| + |F_B|$$

At Equilibrium, $F_L = 0 \Rightarrow$

$$|F_E| = |F_B|$$

$$|eE_H| = |Bev_d| \Rightarrow E_H = Bv_d \quad —①$$

Let, J be the current density of $\bar{e}$'s is given by,

$$J = nev_d \quad —②$$

$$\therefore E_H = \frac{B \cdot J}{ne} \Rightarrow \boxed{E_H = R_H \cdot B \cdot J} \quad —③$$

$$\left(\text{Since } \boxed{R_H = \frac{1}{ne}}\right)$$

R_H is called Hall Coefficient,

If V_H is the Hall voltage in equilibrium, the Hall field is, $E_H = \frac{V_H}{d} \Rightarrow V_H = E_H \cdot d$

$$V_H = R_H \cdot B \cdot J \cdot d$$

$$V_H = \frac{R_H \cdot B \cdot I \cdot d}{d \times w}$$

$$\boxed{\therefore V_H = \frac{R_H B \cdot I}{w}}$$

By measuring I, B, V_H and w, the Hall coefficient is determined.

From the Hall coefficient, one can find the value of the concentration of the charge carriers using the relation,

$$R_H = \frac{1}{ne}$$

By knowing the n, the mobility of the charge carriers is determined by using the relation.

$$\sigma = ne\mu \Rightarrow \mu = \frac{\sigma}{ne}$$

$$\boxed{\mu = \sigma R_H}$$

Applications:-

i. It is used to find whether the given S.C is n- or p-type.

ii). It is used to find the concentration of carriers and hence, the mobility of carriers.

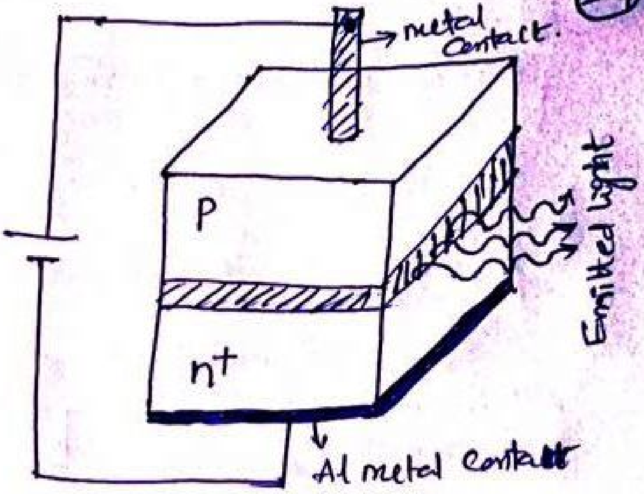
iii) It is used ideally to measure the magnetic field in many applications.

iv). Hall effect S.C devices are used as sensors to sense magnetic fields.

v). used in Electronic switches.

*

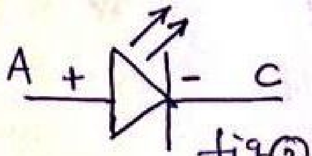
Light Emitting Diode (LED) :-



fig①.

LED is just a normal Semiconductor pn-Junction that works under Forward Bias condition. But these devices Can emitt Radiation by electroluminescence in the UV, visible, or Near IR Regions of the EM. spectrum. The Quanta (photon) of light energy is Released are approximately proportional to the Band gap of the semiconductor

As shown in fig①, the LED is developed on a suitable Substrate. A heavily doped n-type S.C and a p-type S.C are Required to form a pn Junction and aluminium is Required to provide metal contacts.



fig②.

An LED will be just like a normal diode as shown in fig②. under no bias Condition,

* Under forward bias Condition the majority Carriers from both sides of the Junction Can Cross the depletion Region and entering the material at the other side as shown in fig①.
* After Entering the majority carriers into the other side they become minority Carriers.
* For Example, e^-s in n-type (majority) enter the p-type to become minority Carriers.
* The minority Carriers will become larger due to minority carrier Injection.

* Minority Carriers will diffuse and recombine with the majority carriers (holes).

*- The injected e^-'s as minority Carriers in the P-region will recombine with the holes where they are the majority in P-region.

* The recombination process causes light to be emitted and this process is termed as radiative recombination

* To get desired Coloured LED, we must choose the materials with the right Eg, such that

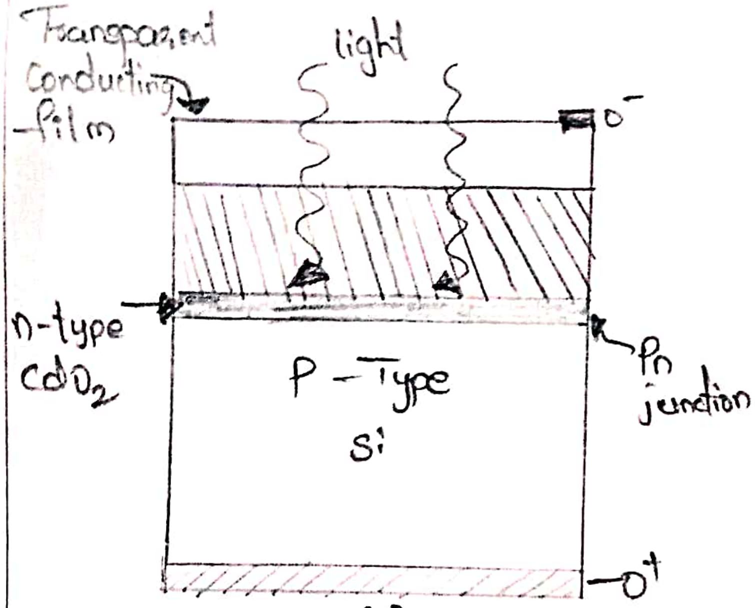
$$\boxed{E_g = \frac{hc}{\lambda}}$$

LED materials :- GaAs — IR
GaP — Red or green
GaAsP — Red or yellow.

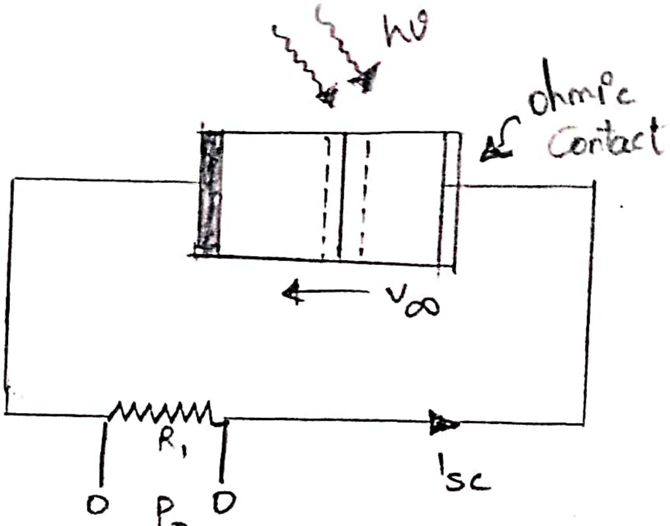
Applications :- i) For Instrument display
ii) Calculators
iii) digital Clocks
iv) For optical switching applications
(V) Light detector
vi) In optical communications

* SOLAR CELL:-

A solar cell is basically a p-n junction that can generate electric power, when illuminated. Solar cells are usually large area devices typically illuminated with sunlight and are intended to convert the solar energy into electrical energy.



(a)

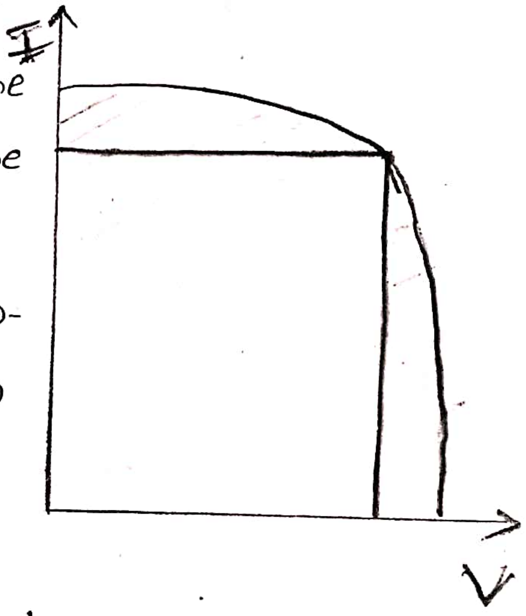


(b)

The Schematic of a Solar cell is shown in fig. It consists of a p-type chip on which a thin layer of n-type material is grown. when the Solar radiation is incident on the cell, electron-hole pairs are generated in the n and p regions. The majority of them cannot recombine in the regions. They reach the depletion region at the junction where an electric filed due to the space charge separates them. Electrons in the p-region are drawn into the n-region and holes in the n-region are drawn into the p-region. It results in accumulation of charge on the two sides of the junction and produces a potential differenced called photo emf. Its magnitude is of the order of 0.5V. The overall power-conversion efficiency of single-crystalline solar cells ranges from 10 to 30%. yielding 10 to 30 mW/cm^2. If a load is connected across the cell a current flows through it. the Sign convention of the current and voltage is shown in fig. It considers a current coming out of the cell to be positive as it leads to electrical power generation.

The power generated depends on the solar cell itself and the load connected to it.

The I-V characteristic of a solar cell is shown in fig we identify the open-circuit voltage, Voc, as the voltage across the illuminated cell at zero current. The short-circuit current, Isc, is the current through the illuminated cell if the voltage across the cell is zero. The short-circuit current is close to the photocurrent while the open-circuit voltage is close to the turn-on voltage of the diode as measured on a current scale similar to that of the photocurrent. the power equals the product of the diode voltage and current and at first increases linearly with the diode voltage but then rapidly goes to zero around the turn-on voltage of the diode. The maximum power is obtained at a voltage labeled as Vm with Im being the current at that voltage.



Solar cells can be connected in parallel or series into solar panels, which can deliver power output of several kilowatts. Solar panels are used in numerous applications in remote locations and in space. Solar cells of all kinds are used in different consumer products- from watches and calculators to power supplies for laptop computers.

MODULE-IV

LASERS

A. LASERS

Introduction:

LASER is a device that emits light through a process of optical amplification based on the Quantum effect called stimulated emission of electromagnetic radiation. LASER is an acronym for Light Amplification by Stimulated Emission of Radiation.

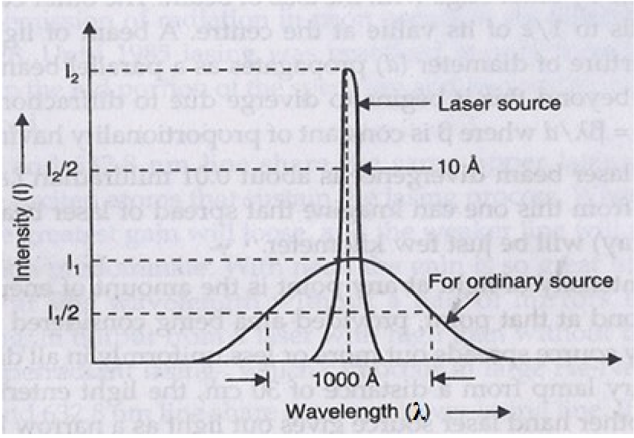
Distinguish between conventional light and Laser light:

Conventional light	Laser light
1. It emits photons in all directions with wide range of wavelengths.	**1.** It emits photons in a narrow, well-defined directional beam.
2. These are incoherent (No fixed phase among the photons emitted by the source).	**2.** These are highly coherent (constant phase relationship among the photons)
3. May be multi-wavelength	**3.** Single wavelength or color.
4. Intensity is low	**4.** Intensity is very high.
5. Ex.:- Electric Bulb, candle, etc.,	**5.** Ex.:- Ruby Laser, He-Ne Laser, etc.,

Characteristics of Laser light

Like ordinary light laser light is electromagnetic in nature. However, there are few characteristics not processed by the normal light. Some of the main characteristics of laser beam are mentioned below:

1. **High Monochromaticity,**
2. **Extremely Directional,**
3. **Extremely Coherence,**
4. **High Intensity and brightness**.



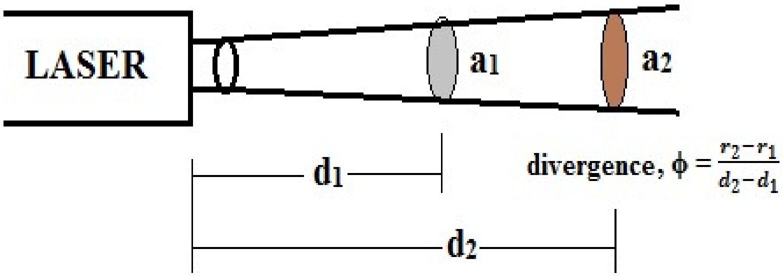
1. Monochromaticity: The light emitted from a laser is highly *monochromatic*, that is, it is of one unique wavelength (color). The light from a laser typically comes from one atomic transition with a **single precise wavelength**. So the laser light has small broadening width ($\Delta\lambda$) and is almost the purest monochromatic light.

For Ordinary white light, $\Delta\lambda$= 300 nm, For monochromatic light, $\Delta\lambda$= 5-10 nm

For Laser light, $\Delta\lambda$= 0.2 nm

2. **Directionality:** We know that when light travels then it tends to spread out, this spreading of light is called divergence and the angle at which the light spreads is called angle divergence (ϕ). Generally divergence is more in ordinary light and lesser in laser light.

   Usually a laser generates less than one mille radian (0.001) divergent beam. This means that a beam from the laser will spread to less than 1cm diameter at a distance of 10 m from the laser. The laser beam divergence is illustrated in the adjacent figure.



3. **Coherence:** The light coming from the Laser is said to be coherent, which means the group of photons (Laser beam) are in phase in space (Spatial) and time (Temporal). Laser light is much more coherent than ordinary light.

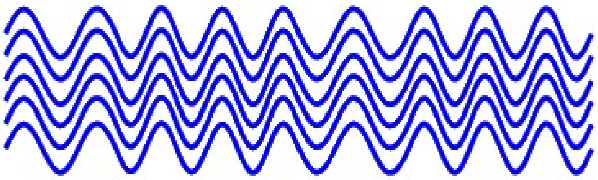


Figure: Coherent Light Waves

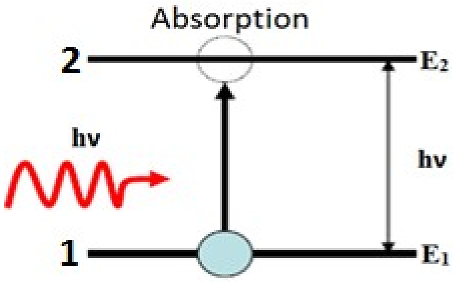
4. **Intensity and Brightness:** Intensity of a wave is defines as energy per unit time flowing through a unit normal area. In Laser light, energy is concentrated in small region of space with small wavelengths with greater intensity. A laser beam has brightness many times in magnitude greater than that of conventional sources due to high directional property of laser beam.

Therefore, Laser light is an extra ordinary light emitted under stimulated and amplified conditions, so that the beam is characterized by high intensity, specific directionality, high monochromaticity and high degree of coherence.

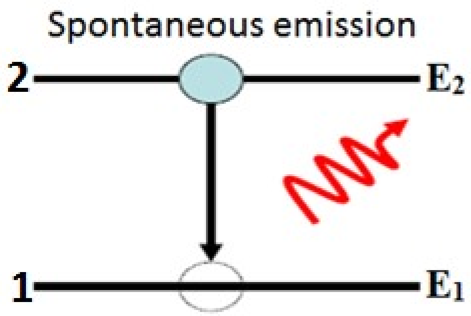
ABSORPTION, SPONTANEOUS AND STIMULATED EMISSION:

To describe the phenomenon of Absorption, spontaneous or stimulated emission, let us consider two energy levels, 1 and 2, of some atom or molecule of a given material, their energies being E_1 and E_2 ($E_1<E_2$).

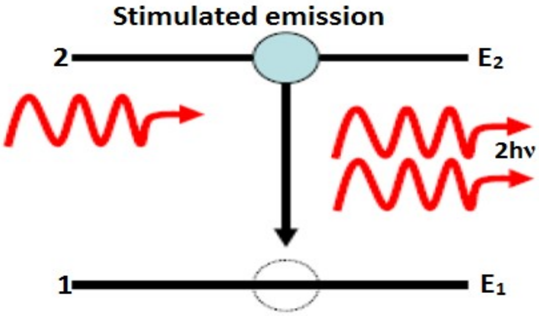
i. **Absorption:** Let us now assume that the atom is initially lying in level 1 (Adj. Fig.). If this is the ground level, the atom will remain in this level unless some external stimulus (photons) is applied to it. We shall assume that, a photon of frequency ν is incident on the material. In this case there is a finite probability that the atom will be raised to level 2. The energy difference $E_2–E_1$ required by the atom to



undergo the transition is obtained from the energy of the incident photon. This is the *absorption* process.



ii. Spontaneous emission: Let us now assume that the atom is initially in level 2. Since $E_2 > E_1$, the atom will tend to decay to level 1. The corresponding energy difference, $E_2–E_1$, must therefore be released by the atom. When this energy is delivered in the form of an electromagnetic wave called photon, the process will be called *spontaneous* (or *radiative*) *emission*.

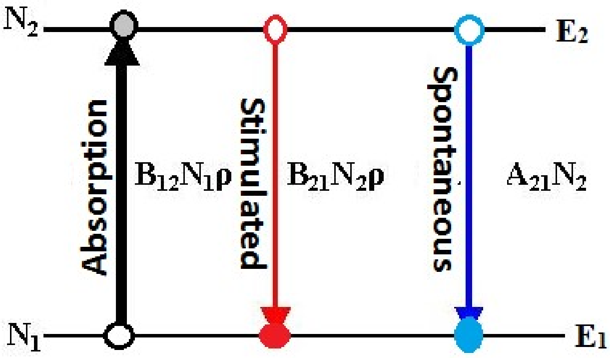


iii. Stimulated emission: Let us now suppose that the atom is found initially in level 2 and that a photon of frequency ν_o is incident on the material (Adjacent Fig.). Since this photon has the same frequency as the atomic frequency, there is a finite probability that this photon will force the atom to undergo the transition 2→1. In this case the energy difference $E_2–E_1$ is delivered in the form of a photon that adds to the incident one. This is the phenomenon of *stimulated emission*.

There is a fundamental difference between the spontaneous and stimulated emission processes. In the case of spontaneous emission, the atoms emits a photon that has no definite phase relation with that emitted by another atom. Furthermore, the photon can be emitted in any direction. In the case of stimulated emission, since the process is forced by the incident photon, the emission of any atom adds in phase to that of the incoming photon and along the same direction.

EINSTEIN'S COEFFICIENTS:

In 1916, **Albert Einstein** proposed that there are three processes occurring in the formation of an atomic spectral line. The three processes are referred to as **spontaneous emission, stimulated emission, and absorption**. With each is associated an Einstein coefficient which is a measure of the probability of that particular process occurring. Einstein considered the case of isotropic radiation of frequency *v*, and **energy density *ρ*(*v*)**.



MODULE-IV

Let us consider N_1 and N_2 be the populations in the energy levels of energies E_1 and E_2 respectively in a system of atoms at a thermal equilibrium of temperature T.

Upward transition:

Absorption is the process by which a photon is absorbed by the atom, causing an electron to jump from a lower energy level E_1 to a higher one E_2. The process is described by the Einstein coefficient B_{12},

The absorption rate is directly proportional to N_1 and $\rho(\nu)$

Therefore, $Rate\ of\ absorption = B_{12}N_1\rho(\nu)$ **------------- (1)**

Here B_{12} is **Einstein's coefficient of absorption**.

Downward transition:

Spontaneous emission is the process by which an electron "spontaneously" (i.e. without any outside influence) decays from E_2 to E_1. The process is described by the Einstein coefficient A_{21}.

Spontaneous emission rate is directly proportional to N_2 only.

Therefore, $Rate\ of\ spontaneous\ emission = A_{21}N_2$ **------------- (2)**

Here, A_{21} is the **Einstein's coefficient of spontaneous emission**.

Stimulated emission is the process by which an atomic electron in the excited E_2 is interacting with a photon of certain frequency may drop to a lower energy level E_1, transferring its energy to that photon. A new photon created in this manner has the same phase, frequency and direction of travel as same as the incident photon. The process is described by the Einstein coefficient B_{21}.

Stimulated emission rate is directly proportional to N_2 and $\rho(\nu)$.

Therefore, $Rate\ of\ stimulated\ emission = B_{21}N_2\,\rho(\nu)$ **---------- (3)**

Here, B_{21} is the **Einstein's coefficient of stimulated emission**.

Consider an ideal material with only two non-degenerate energy levels, at thermal equilibrium,

Absorption = spontaneous emission + Stimulated emission **------ (4)**

i.e. $B_{12}N_1\rho(\nu) = A_{21}N_2 + B_{12}N_2\rho(\nu)$ **------- (5)**

$B_{12}N_1\rho(\nu) - B_{12}N_2\rho(\nu) = A_{21}N_2$

$$[B_{12}N_1 - B_{21}N_2]\,\rho(\nu) = A_{21}N_2$$

$$\rho(\nu) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \qquad \text{or} \quad \rho(\nu) = \frac{A_{21}N_2}{B_{21}N_2\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}}$$

or

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \qquad \text{----------------- (6)}$$

But, in thermal equilibrium, the Boltzmann's distribution law applies, so,

$$N_1 = N_0 exp\left(\frac{-E_1}{k_BT}\right) \; and \; N_2 = N_0 exp\left(\frac{-E_2}{k_BT}\right)$$

where, N_o is population in ground state and k_B is the Boltzmann's constant.

Therefore, $\frac{N_1}{N_2} = \exp\left[\frac{E_2-E_1}{k_BT}\right] = \exp\left[\frac{h\nu}{k_BT}\right]$ (Since, E_2-E_1= $h\nu$) --------- (7)

Substituting eq. (7) in eq. (6), we get

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\frac{1}{\left\{\left[exp\left(\frac{h\nu}{k_BT}\right)\left(\frac{B_{12}}{B_{21}}\right)\right]-1\right\}} \qquad \text{-------------- (8)}$$

According to Plank's law of black body radiation at temperature T we have the energy density $\rho(\nu)$ at frequency ν is

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\left\{exp\left(\frac{h\nu}{k_BT}\right)-1\right\}} \qquad \text{-------------- (9)}$$

Comparing eq. (8) and eq. (9), we get

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{----- (10)}} \qquad \boxed{\frac{B_{12}}{B_{21}} = 1 \text{ -------- (11)}}$$

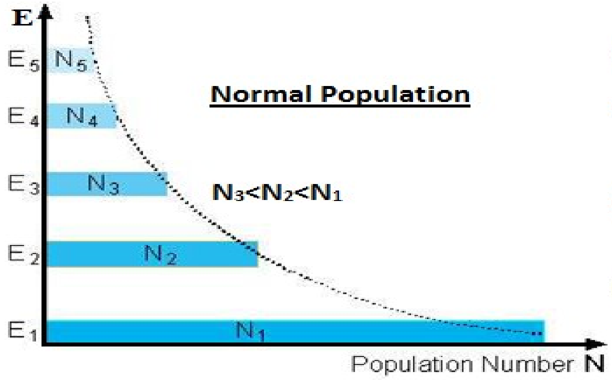
Equations (10) and (11) show the relations between Einstein's coefficients B_{12}, B_{21} and A_{21}.

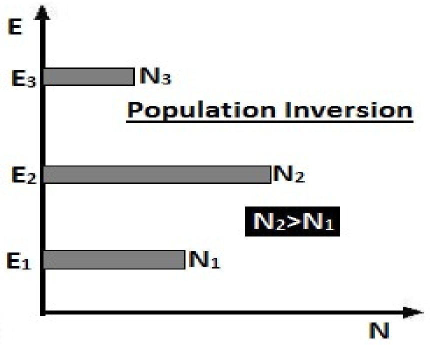
From the above relation eq. (10) shows that the ration of rate of spontaneous (A_{21}) to rate of stimulated emission (B_{21}) is **proportional to cube of the frequency** of the incident radiation. The eq. (11) shows that the rate of absorption is equal to the rate of stimulated emission, when the system in equilibrium.

POPULATION INVERSION:

At thermal equilibrium, the no of atoms in level 2 is less than the no of atoms in level 1 ($N_2<N_1$). **The process of making of higher population level 2 than the population in level 1 is**

known as population inversion ($N_2>N_1$) **This is highly non equilibrium situation**. When the population inversion exists, rate of stimulated emission is greater than rate absorption occurs and the light passing through the material is amplified.





LASER PUMPING:

The process of achieving the population inversion by sending the atoms from lower level to higher levels through supply of energy to the lower level atoms is called **laser pumping**. The supply of energy is usually provided in the form of **light (Optical) or electric current (Electrical)**. But, more exotic sources such as chemical or nuclear reactions can also be used.

i. **Optical Pumping:** Optical pumping is a process in which light is used to raise or pump electrons from lower energy level in an atom or molecule to higher one. It is commonly used in solid state lasers (Ex.: Ruby Laser, etc.).
ii. **Electrical Pumping:** Electric discharge is common in gas lasers. Gas lasers have very narrow absorption band, pumping then in any flash lamp is not possible. In most of the cases population inversion is created by means of electric discharge. (Ex.: He-Ne Laser etc.)

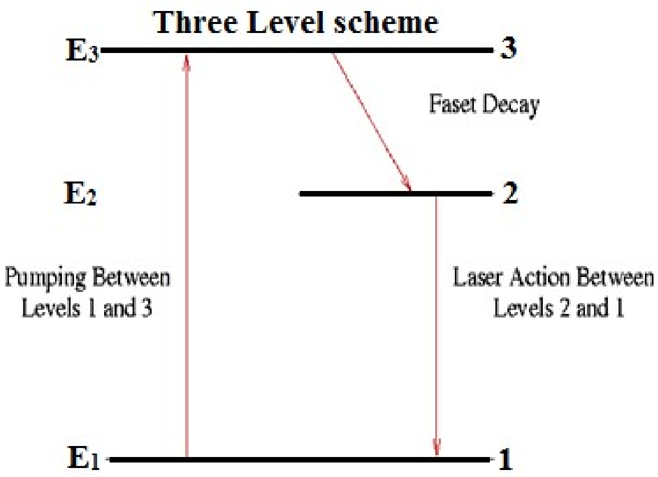
META-STABLE STATE:

Metastable state is an excited state of an atom or other system with a longer lifetime than the other excited states. However, it has a shorter lifetime than the stable ground state. Atoms in the metastable state remain excited for a considerable time in the order of $10^{-6}$to 10^{-3}s. During metastable state, all the parameters associated with state hold stationary values. A large number of excited atoms are accumulated in the metastable state.

The population of metastable state can exceed the population at a lower level thereby establishing population inversion in a lasing medium. Population inversion could not be created without a metastable state.

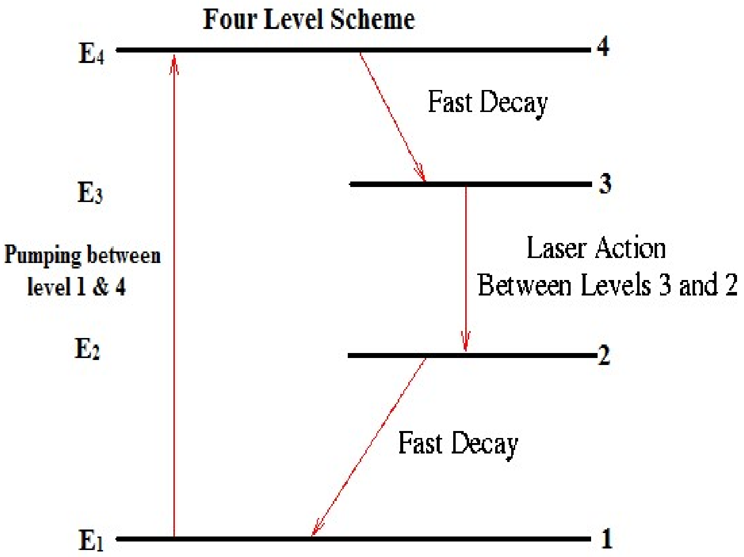
THREE LEVEL SCHEME:

We have seen that the key to laser action is to obtain a population inversion between two levels of energy E_1 and E_2 with $E_2>E_1$, so that more atoms are in the level 2 than in the level 1. In the three-level lasers figure, we look for three levels in an atom such that $E_3>E_2>E_1$, with a fast decay between levels 3 and 2 and a slow decay between 2 and 1. Incident radiation of angular frequency $\nu_{31} = (E_3 - E_1)/\hbar$ is used to raise as many atoms as possible from the level 1 to level 3. Note that a population inversion cannot be obtained between levels 3 and 1, because when the number of atoms N_3 in level 3 equals the number N_1 in level 1, absorption will be balanced by stimulated emission and the material will become transparent at the frequency ν_{31}. If level 3 decays rapidly to level 2, a population inversion can be obtained between levels 2 and 1. An example is the ruby laser.



FOUR LEVEL SCHEME:

Except in special cases, such as the ruby, it is difficult to produce a population inversion between a ground state and an excited state, because initially all the atoms are likely to be in the ground state, and we have to get more than half the atoms into level 2 before a population inversion can be achieved. An easier approach is to use a four-level system in the adjacent figure and attempt to create a population inversion between two excited levels. We start with all the atoms in the ground state1, and none in the excited states 2, 3 and 4 ($E_2<E_3<E_4$). Level 4 is chosen so that it has a fast decay to level 3, and pumping between levels 1 and 4 immediately produces a population inversion between levels 3 and 2. As level 2 begins to fill up by stimulated emission at the frequency $(E_3-E_2)/h$, the population inversion will decrease. To minimize this, level 2 is chosen so that it has a fast decay to the ground state.



REQUIREMENTS FOR THE LASER SYSTEM:

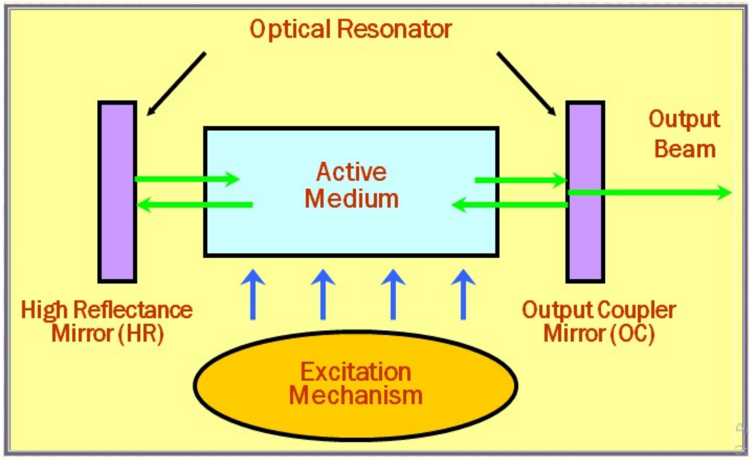
The basic requirements of the Laser system are shown in below figure. It consists of three basic parts as given below:

i. Pumping System
ii. Active medium
iii. Optical Resonator

i. **Pumping System:** Pumping source is the basic energy source for a laser. It gives energy to various atoms of laser medium & excites them, so that population inversion can take place and it is maintained with time. The excitation of atom occurs directly or through atom or atom collision.

There are various types of pumping systems depending on the nature of the active medium.

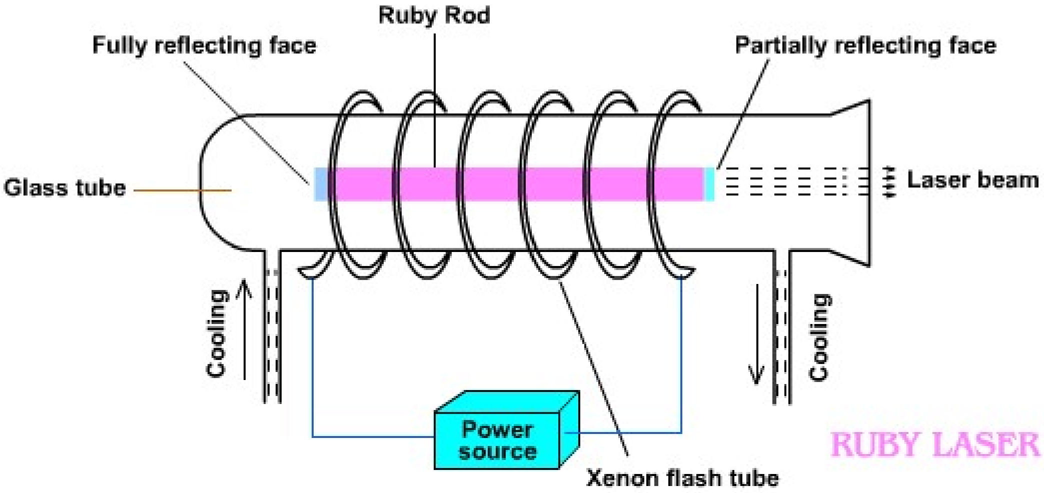
ii. **Active Medium:** Active medium is the heart of the laser system and is responcible for producing population inversion (gain) and subsequent generation of laser. It can be a solid, liquid, semiconductor or gas medium and can be pumped to higher energy state.



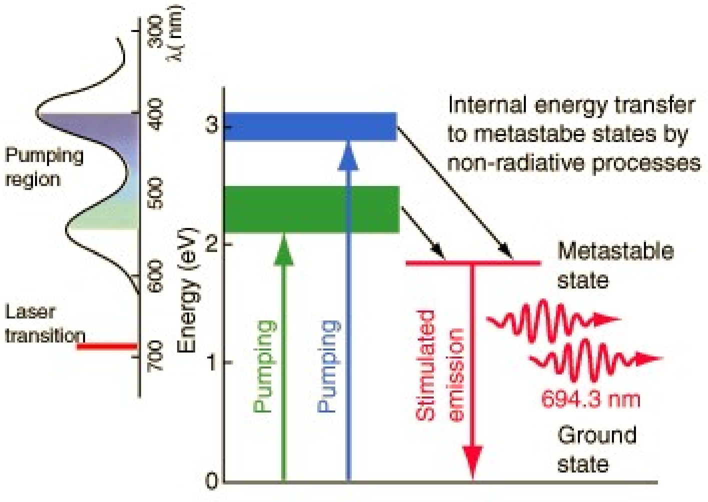
iii. **Optical resonator:** It plays a very important role in the production of laser beam from the laser system. It is a setup used to obtain amplification of slimulated photons, by oscillating them back and forth between two extreme limits. It consists of two plane or concave mirrors placed co-axially. One mirror is fully reflecting and other is partially reflecting.

RUBY LASER:

The first working laser was built in 1960 by T.H. Maiman using a Ruby crystal and so called the Ruby Laser. This is also called **solid state laser or three level laser**. Ruby belongs to the family of gems consisting of Al_2O_3 with various types of impurities. For example in Ruby laser used the pink ruby contains 0.05% Cr atoms (Al_2O_3+0.05%Cr_2O_3). The schematic diagram of the Ruby laser is as follows:



Construction: The Ruby laser consists of a ruby rod whose length is few centimeter and diameter is 0.5 cm. This is made up of with chromium (Cr^{3+}) doped Al_2O_3 material. Both the ends of the ruby rod are silvered such that one end is fully reflecting and the other end is partially reflecting. The ruby rod is surrounded by helical xenon flash lamp tube which provides the optical pumping to raise Cr^{+3} ions to upper energy level. The light from the xenon flash tube was focused by the cylindrical cavity onto the ruby rod, thereby exciting the chromium atoms which were responsible for the laser action.



Working: The ruby laser is a three-Level system (Adjacent Fig.) since only three energy levels are involved in the process of stimulated emission. The depopulation of the ground state for population inversion is achieved by exciting the atoms of the ruby crystal with intense light from a xenon flash lamp. Thus the atoms are excited from the ground state (level 1) to an upper state (level 3) by means of absorption. From the energy level 3, the atoms are transferred to energy level 2 without emitting radiation (non-radiative transfer). The energy level 2 is called met stable level since the atoms stay at this level for a longer interval of time. Finally, the atoms return to the ground state from the meta stable level through the process of stimulated emission giving rise to an intense laser light at **$6943A^{o}$**. The laser beam comes out in the form of a pulse of very short duration (about a millisecond).

Applications: They are still used in a number of applications where short pulses of red light are required.

i. Holography's around the world produce holographic portraits with ruby lasers, in sizes up to a meter squared.
ii. Many non-destructive testing labs use ruby lasers to create holograms of large objects such as aircraft tires to look for weaknesses in the lining.
iii. Ruby lasers were used extensively in **tattoo and hair removal**.

Drawbacks:

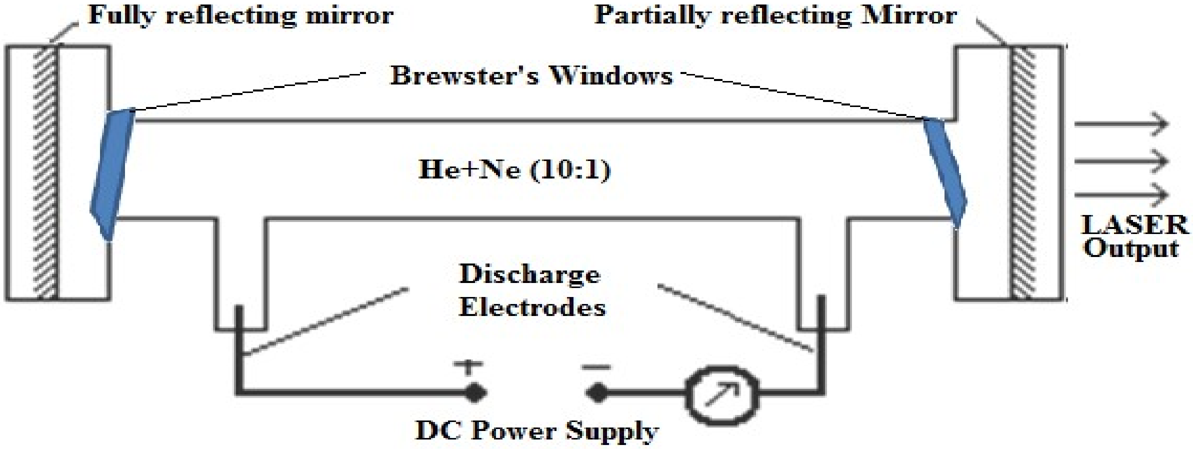
i. The laser requires high pumping power because the laser transition terminates at the ground state and more than half of ground state atoms must be pumped to higher state to achieve population inversion.
ii. The efficiency of ruby laser is very low because only green component of the pumping light is used while the rest of components are left unused

iii. The laser output is not continues but occurs in the form of pulses of microseconds duration.

HELIUM-NEON (He-Ne) LASER:

The He-Ne laser active medium consists of two gases which do not interact form a molecule. Therefore He-ne laser is one type of **atomic gas laser** and also called as **Four level laser**.

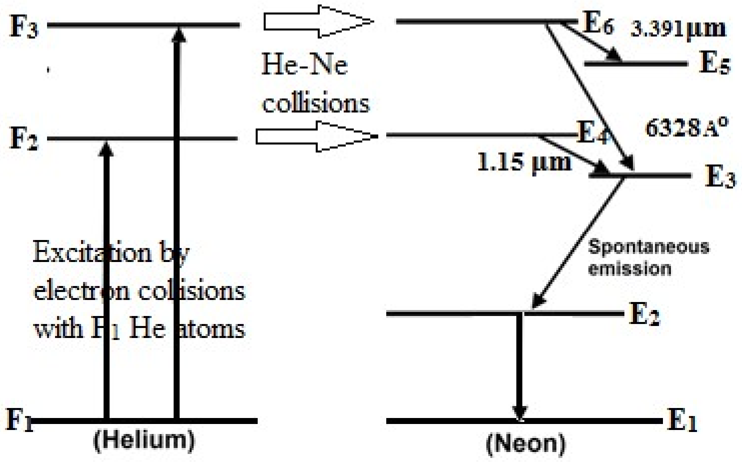
Construction:



He - Ne gas laser consists of a gas discharge tube of length 30cm and diameter of 1.5cm. The tube is made up of quartz and is filled with a mixture of Neon under a pressure of 0.1mm of Hg. The Helium under the pressure of 1mm of Hg, the ratio of He-Ne mixture of about 10:1, hence the no. of helium atoms are greater than neon atoms. The mixtures is enclosed between a set of parallel mirrors forming a resonating cavity, one of the mirrors is completely reflecting and the other partially reflecting in order to amplify the output laser beam.

Working:

In the He-Ne laser the light produced by atomic transitions within the Neon atom. The Helium does not directly produce laser light but it acts as a buffer gas, this purpose of which is to assist/help the atoms of the other gas to produce lasing action.



The active energy levels of He and Neon atoms are show in adjacent figure. In helium there are three active energy levels named as F_1, F_2 and F_3 where as in Neon, there are six active energy levels named as E_1, E_2, E_3, E_4, E_5, and E_6. In Helium, the metastable states are F_2 and F_3, where as in Neon, E_4 and E_6. When a discharge is passed through the gaseous mixture electrons are accelerated in the tube these accelerated electrons collide with the helium atoms and excite them to higher energy levels (F_2 and F_3) since the levels are metastable energy levels, He atoms spend sufficiently long time. Inelastic collision of the excited helium atoms (F_2 and F_3) with the ground state (E_1) Neon atoms results in transfer of energy to the neon atoms, exciting them into E_4 and E_6 states. This is due to the coincidence of energy levels between the helium and neon atoms. Since E_4 and E_6 are metastable states, hence the population inversion takes place between the E_6 and E_4 with respect to E_5 and E_3. The stimulated emission takes place between $E_6 \rightarrow E_3$ gives a laser light of wave length **6328A^o** and the stimulated emission between $E_6 \rightarrow E_5$ gives a laser light wavelength of **3.39µm**. Another stimulated emission between $E_4 \rightarrow E_3$ gives a laser light wavelength of **1.15µm**. The neon atoms undergo spontaneous emission from $E_3 \rightarrow E_2$ and $E_5 \rightarrow E_2$. Finally the neon atoms are returned to the ground state E_1 from E_2 by non-radiative diffusion and collision process.

After arriving the ground state, once again the neon atoms are raised to $E6$ & $E4$ by excited helium atoms thus we can get **continuous wave** output from He-Ne laser. But some optical elements placed insides the laser system are used to absorb the infrared laser wavelengths 3.39µm and1.15µm. Hence the output of He-Ne laser contains only a single wavelength of 6328A^o and the output power is about few milliwatts.

Applications:

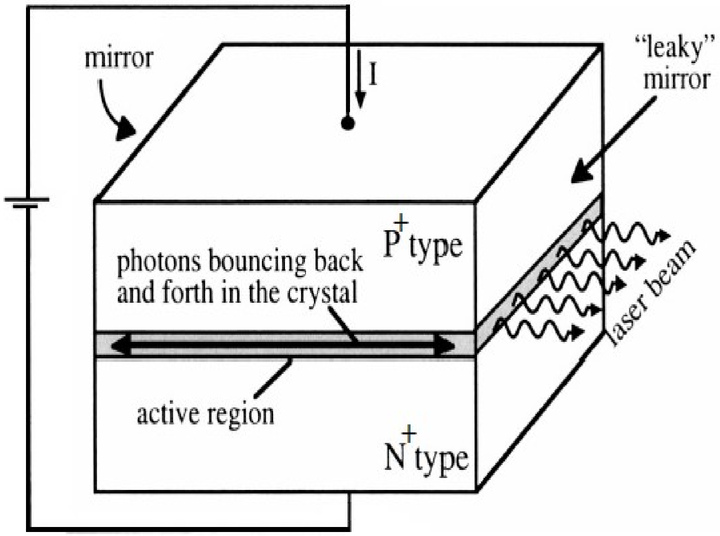
i. The Narrow red beam of He-Ne laser is used in supermarkets to read bar codes (Bar-code scanners).
ii. The He-Ne Laser is used in Holography in producing the 3D images of objects.
iii. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

SEMICONDUCTOR (Diode) LASER (GaAs Laser):

Semiconductor lasers also known as *quantum well lasers* are smallest, cheapest, can be produced in mass, and are easily scalable. They are basically p-n junction diode, which produces light of certain wavelength by recombination of charge carrier when forward biased, very similar to the light-emitting diodes (LEDs). LEDs possess spontaneous emission, while laser diodes emit radiation by stimulated emission.

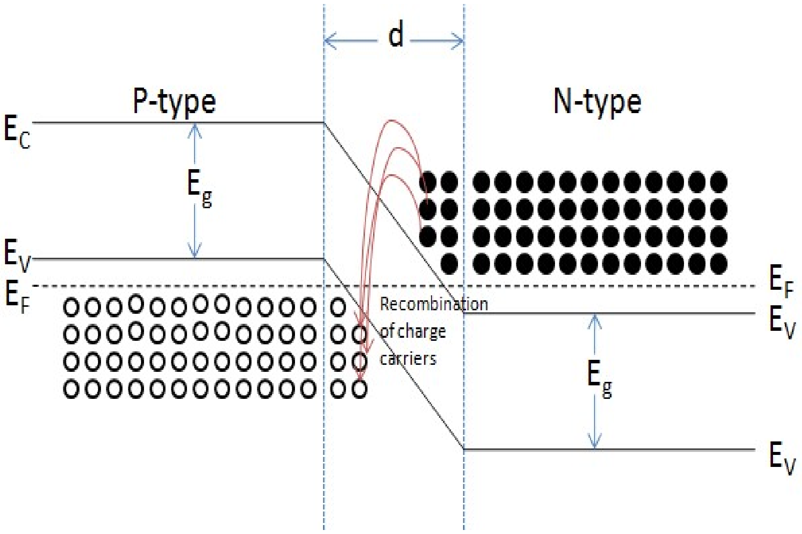
Principle: In the case direct band gap semiconductors there is a large possibility for direct recombination of hole and electron emitting a photon. GaAs is a direct band gap (1.44 eV) semiconductor and hence it is used to make lasers and light emitting diodes. The wave lengths of the emitted light depend on the band hap of the material.

Construction:



The P^+ and N^+ regions of the diode are obtained by heavily doped p- and n-regions of GaAs. The thickness of the p-n junction layer is very narrow at the junction, the side walls are well polished and parallel to each other. Since the refractive index of GaAs is high, the reflectance at the material air interface is sufficiently large so that the external mirrors are not necessary to produce multiple reflections. The p-n junction is forward biased by connecting positive terminal to p-type and negative terminal to n-type.

Working:



The population inversion can be obtained by injecting electrons and holes in to the junction from the n-region and p-region by means of forward bias voltage. When the forward bias is not connected, no electrons and holes present in the depletion region. When small forward bias voltage is given to the p-n junction then small number of electrons and holes will injected into the depletion region from respective regions. When relatively a large current of the order of 10^4 A/cm^2 is passed through the junction then large number of electrons and holes will be injected into the depletion region as shown in above figure. Then the direct recombination processes take place between holes and electrons in the depletion region and release the photons. Further the emitted photons increase the rate of recombination. Thus more number of photons produced having same phase and frequency of the induced photons.

The wavelength of the emitted radiation depends on the energy band gap of the semiconductor material. The energy gap of the GaAs Semiconductor is 1.44 eV then it emits laser light of wavelength ~8600 A^o.

$$\lambda = \frac{hc}{E_g} = \frac{6.625\times10^{-34}\times3\times10^{8}}{1.44} = 8626\ \mathrm{A^o}$$

The efficiency of the laser emission increases when a cooling arrangement provided to the laser diodes.

APPLICATIONS OF LASER: There are many scientific, military, medical, industrial and commercial laser applications which have been developed since the invention of the laser.

i. **Laser Drilling:** Laser drilling is one of the few techniques for producing holes in hard materials with desired diameter. The diameter of these holes as small as 5 microns.
Laser-drilling is used in many applications, including the oil gallery of some engine blocks, aerospace turbine-engine cooling holes, laser fusion components and printed circuit board micro-vias.

ii. **Laser Welding and Cutting:** The highly collimated beam of a Laser can be further focused to a microscopic dot of extremely high energy density for welding and cutting. The automobile industry makes extensive use of CO_2 laser with powers up to several KW for computer controlled welding on auto assembly lines.

iii. **Lasers in Data Storage:** The reading and writing (Store) of the data on a compact disc (CD or DVD) is done with semiconductor laser.

iv. **Lasers in scientific research:**
  a) Lasers are used to clean delicate pieces of art, develop hidden finger prints
  b) Laser are used in the fields of 3D photography called holography
  c) Using lasers the internal structure of micro organisms and cells are studied very accurately
  d) Lasers are used to produce certain chemical reactions.

v. **Laser in Medicine:**
  a) The heating action of a laser bean used to remove diseased body tissue
  b) Lasers are used for elimination of moles and tumours, which are developing in the skin tissue.
  c) Laser beam is used to correct the retinal detachment by eye specialist.

vi. **It is used in biomedical science.**

vii. **It is used in 3D photography.**

viii. **It can be used for measuring velocity of light, to study spectrum of matters, to study Raman effect.**

ix. **It is used in holography.**

x. **It is also used in military, like LIDAR.**

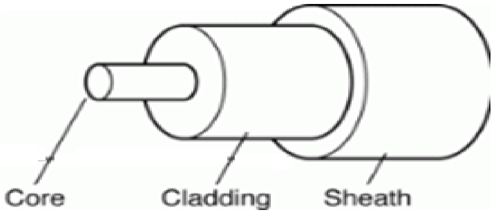
xi. **It is used to accelerate some chemical reactions.**

xii. **It is also used in nuclear fusion reactions.**

B. FIBER OPTICS

STRUCTURE OF AN OPTICAL FIBER:

Optical Fiber: A very thin, long, flexible, transparent, cylindrical dielectric medium which guide the light signal propagated through it.



It consists of three parts:

1. The core
2. The cladding,
3. The outer jacket.

- The core is the inner part of the fiber, which guides the light signal.
- The cladding surrounds the core completely.
- The refractive index of the core (n_1)is greater than the cladding(n_2) to satisfy the total internal reflection ($n_1 > n_2$)
- The outer jacket provides the mechanical protection to the fiber.

The core and cladding diameters are about few microns. Optical fibers are arranged in bundles called optical cables and used to transmit light signals over long distances.

PRINCIPLE AND WORKING OF OPTICAL FIBER:

Principle: The transmission of light in optical fiber is based on the principle of **Total Internal reflection**.

Let n_1 and n_2 be the refractive indices of core and cladding respectively such that $n_1>n_2$. Let a light ray travelling from the medium of refractive index n_1 to the refractive index n_2 be incident with an angle of incidence θ and the angle of refraction r. By Snell's law,

$$n_1 \sin\theta = n_2 \sin r \qquad \text{----------} \ (1)$$

The refracted ray bends towards the normal as the ray travels from rarer medium to denser medium. On the other hand, the refracted ray bends away from the normal as it travel from denser medium to rarer medium. In Optical fiber, the light ray travelled from core (denser) to cladding (rarer) medium, there is possibility of total internal reflection, if the angle of incidence is greater than the critical angle (θ_c).

Critical Angle: When a light ray moves from high refractive index (core) medium to low refractive index (cladding) medium and for a particular angle of incidence the refraction angle (r) is 90^o then the angle of incidence is known as critical angle (θ_c).

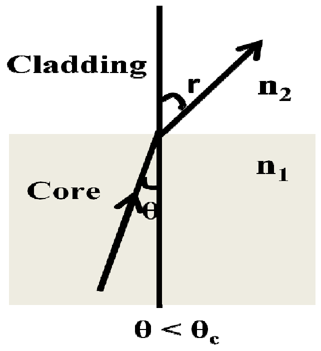


Fig.1

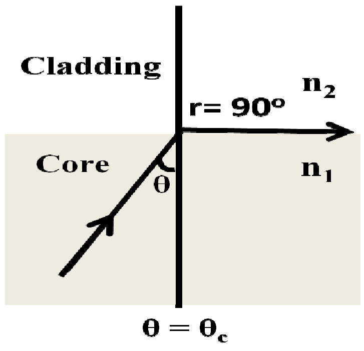


Fig.2

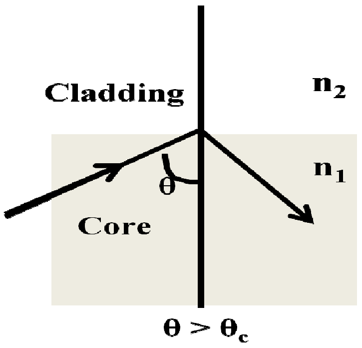


Fig.3

i. When $\theta < \theta_c$, then the ray refracted into the second medium as shown in above fig.1.
ii. When $\theta = \theta_c$, then the ray travels along the interface of two media as shown fig. 2.
iii. When $\theta > \theta_c$, then the ray totally reflects into the same medium (core) as shown in fig.3.

The critical angle can calculated from Snell's law, suppose if $\theta = \theta_c$ then $r = 90^o$, hence

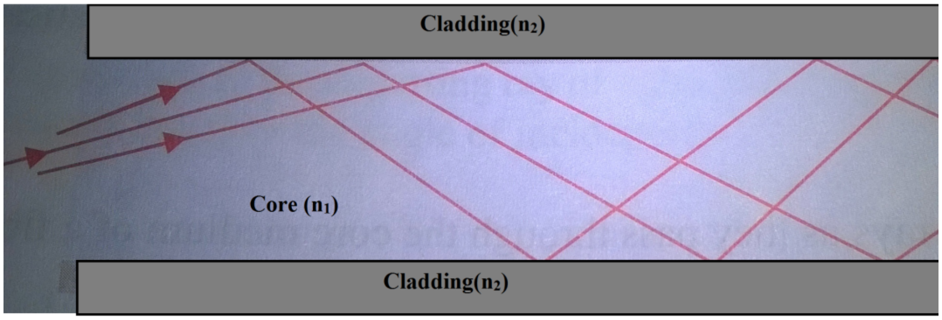
$$n_1 \sin \theta_c = n_2 \sin 90^o$$

$$sin\theta_c = \frac{n_2}{n_1}$$

or $$\boldsymbol{\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

This is known as critical angle of mediums of refractive indices n_1 and n_2 ($n_1 > n_2$).

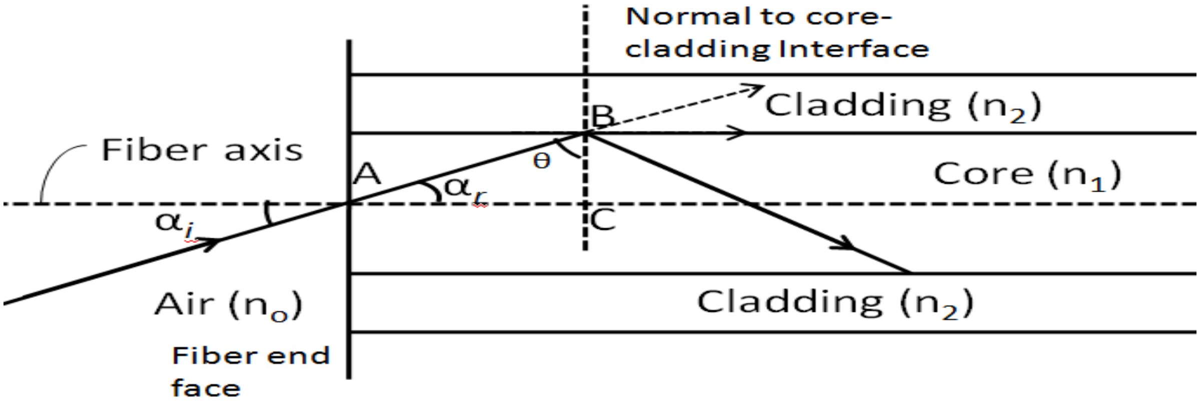
The light signal once entered the fiber and takes total internal reflection with in the core as shown in fig.4, it will continue to propagate till the other end of the fiber.



ACCEPTANCE ANGLE:

The maximum angle of incidence to launch the beam at its one end to enable the entire light to pass through the core is called **acceptance angle**.

When we launch the light beam in to the fiber at its one end the entire light may not pass through the core and propagate. Only the rays which make the angle of incidence greater than critical angle undergo total internal reflection and propagate through the core and all other rays are lost. Let us consider a ray enters the core of refractive index $\boldsymbol{n_1}$ from air medium of refractive index $\boldsymbol{n_o}$ with an angle of incidence $\boldsymbol{\alpha_i}$ at the interface of air and core and incident at the interface of core and cladding with an angle of incidence $\boldsymbol{\theta}$ as shown in below figure.



If the angle of incidence at the interface of air and core is maximum ($\boldsymbol{\alpha_i = \alpha_{max}}$), the angle of incidence at the interface of core and cladding is equal to critical angle ($\boldsymbol{\theta = \theta_c}$). If the angle α_i is less than the α_{max}, the angle of incidence at the interface of core and cladding would be greater than the critical angle ($\theta > \theta_c$), and further total internal reflections cascades through the fiber.

$$From\ fig., \Delta ABC,$$

$$\alpha_r = 90 - \theta$$

$$At\ air - core\ interface, the\ Snell's\ law\ is, \quad \frac{\sin \alpha_i}{\sin\ \alpha_r} = \frac{n_1}{n_o}$$

$$\sin \alpha_i = \frac{n_1}{n_o} \sin \alpha_r$$

$$\sin \alpha_i = \frac{n_1}{n_o} \sin(90 - \theta)$$

$$\sin \alpha_i = \frac{n_1}{n_o} \cos \theta$$

$$\boldsymbol{\sin \alpha_{max} = \frac{n_1}{n_o} \cos\ \theta_c} \text{ (When } \alpha_i = \alpha_{max},\ \theta = \theta_c) \qquad \text{----- (1)}$$

Snell's law at core and cladding interface gives,

At critical angle($\theta=\theta_c$), the angle of refraction is 90^o

$$n_1 \sin \theta_c = n_2 \sin 90$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\cos \theta_c = \sqrt{1 - sin^2\theta_c} = \sqrt{1 - \frac{n_2^2}{n_1^2}} \qquad \text{------ (2)}$$

From Eq. (1), we get, $\boldsymbol{\sin \alpha_{max} = \frac{n_1}{n_o}\sqrt{1 - \frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}}$

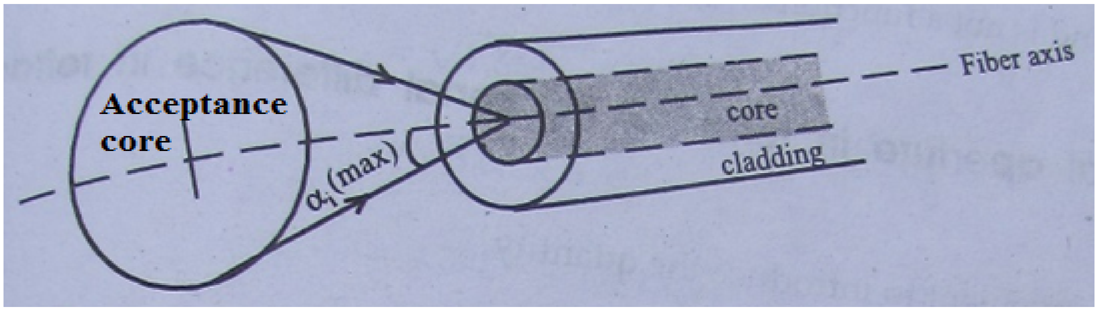
Thus,

$$\boldsymbol{\sin \alpha_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}}$$

$$\boldsymbol{\alpha_{max} = sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_o}\right)} \qquad \text{----- (3)}$$

This $\boldsymbol{\alpha_{max}}$ is known as **Acceptance angle.**

ACCEPTANCE CONE: The acceptance cone is derived by rotating the Acceptance Angle about the fiber axis. Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection. Larger acceptance angles make launching easier.



Numerical Aperture: The numerical aperture (*NA*) of an optical fiber is defined as sin of acceptance angle and is dimensionless number that characterizes the range (ability) of angles over which the system can accept light.

Therefore, $$\boldsymbol{NA = \sin(\alpha_{\max})} = \frac{\sqrt{n_1^2-n_2^2}}{n_o} \qquad \text{-------- (4)}$$

If the refractive index of air is unity (μ_{air}=1), thus the Eq. (4) reduces as,

$$NA = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta} \qquad \text{where } \Delta = \frac{n_1 - n_2}{n_1} \text{ called Fractional Index difference}$$

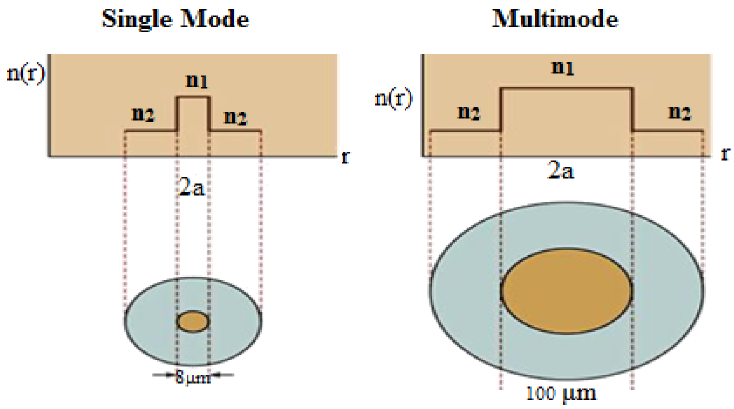
FIBER TYPES AND REFRACTIVE INDEX PROFILES:

Depending upon the refractive index profile of the core, optical fibers are classified into two types. They are:

i. Step Index (SI) Fiber ii. Graded Index (GI) fiber

i. Step Index (SI) Fiber: In step index fibers, the refractive index of the core is uniform throughout the medium and undergoes an abrupt (Step) change at the interface of core and cladding.

The light in the fiber propagates by bouncing back and forth from core-cladding interface. The step index fibers propagate both single and multimode signals within the fiber core. The light rays propagating through it are in the form of meridinal rays which will cross the fiber core axis during every reflection at the core – cladding boundary and are propagating in a zig – zag manner.



The variation of refractive index profile with radial distance *r* for the SI fiber is shown in the above figure.

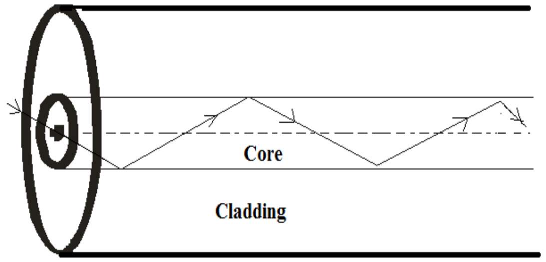
$$\begin{aligned} n(r) &= \boldsymbol{n_1}(\text{core}) \\ &= \boldsymbol{n_2}\ (\text{cladding}) \end{aligned}$$

The number of possible propagation modes in the core depends on the radius of the core and NA of the fiber and is given by V-number as

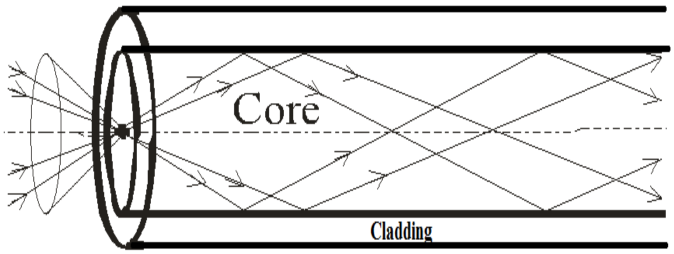
$$\boldsymbol{V = \frac{2\pi}{\lambda} a(NA)}$$ Where '*a*' is the radius of the core and NA-Numerical Aperture.

The number of modes (paths) through the SI fiber is $= \boldsymbol{\frac{V^2}{2}}$

The propagation modes in the SI Fiber in both the modes are shown below:



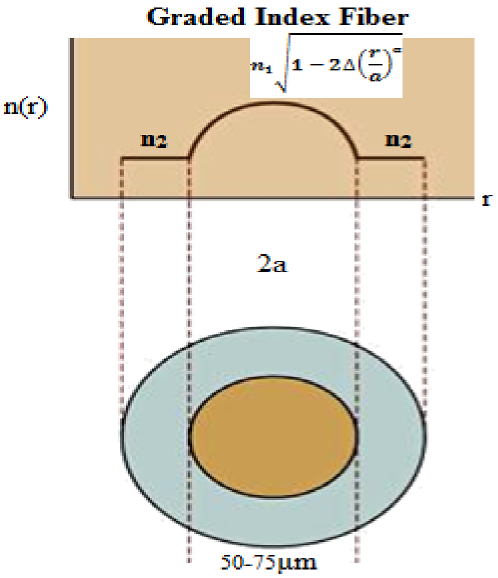
Single mode SI fiber



Multimode SI Fiber

ii. **Graded Index (GI) Fiber:** In graded index fibers, the refractive index of the core varies gradually as a function of radial distance from the fiber center.

The refractive index of the core decreases as we move away from the centre. The refractive index of the core is made to vary in the form of parabolic manner such that the maximum refractive index is present at the centre of the core.



The variation of refractive index profile with radial distance *r* for the SI fiber is shown in the adjacent figure.

$$n(r) = n_1\sqrt{1-2\Delta\left(\frac{r}{a}\right)^{\alpha}}\ (0 \le r \le a)\ \text{(core)}$$

$$= n_2\ \text{(cladding)}$$

Here α is the grading parameter which decides the variation of RI in core.

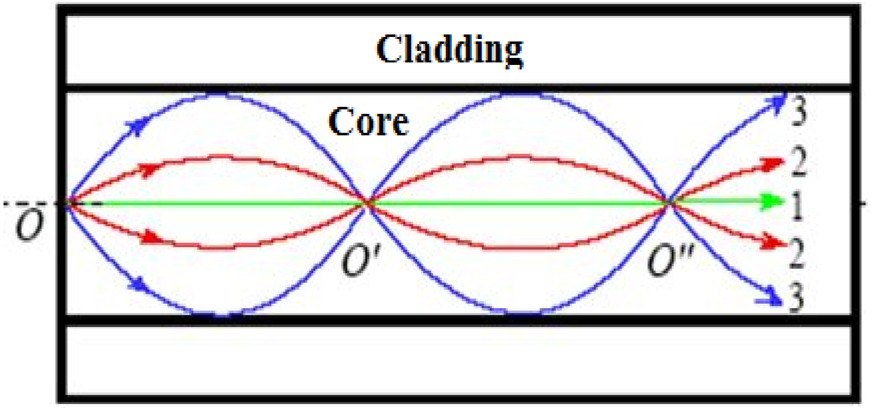
α = 1 for linear grading,
α =2 for **parabolic** grading
α = ∞ for Step Index grading.

The transmitted light signals travel through the core medium in the form helical (sine waves) rays, which will not cross the fiber axis at any time.

The number of modes propagated through the GI Fiber depends on the radius of the core and NA of the fiber. Therefore,

The possible number of modes propagated through the GI Fiber is $= \frac{V^2}{4}$

The propagation modes in the GI Fiber are shown below:



Graded Index Fiber

ATTENUATION:

Attenuation means loss of light energy as the light pulse travels from one end of the fiber cable to the other. It is also called as signal loss or fiber loss. It is directly proportional to the length of the cable. It limits the optical power which can reach the receiver at the destination end of the fiber.

Attenuation is mainly caused as a result of

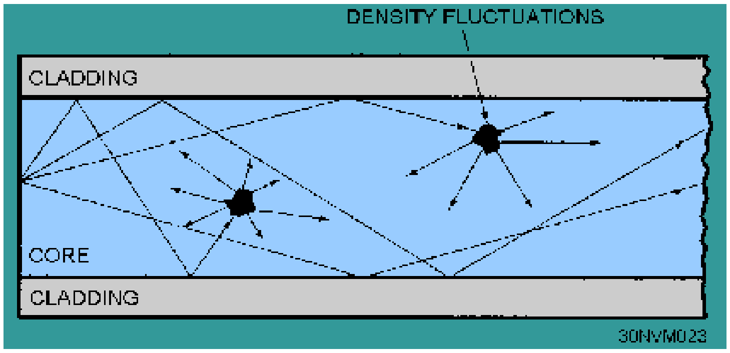
i. Absorption loss
ii. Scattering loss and
iii. Bending losses.

Attenuation is defined as the ration of input optical power (P_i) to the output optical power (P_o) The following equation defines the signal attenuation as a unit of length:

$$\alpha(dB/km) = -\frac{10}{L}\log_{10}\left(\frac{P_O}{P_i}\right)$$

i. Absorption loss: Every material has a characteristic of absorbing a fraction of the incident light. Optical fibers are also no exception. This property is called intrinsic absorption. Besides the intrinsic absorption, the impurities whatsoever present in optical fiber also absorb light which is called impurity (Extrinsic) absorption. Such types of absorptions result in the reduction of the strength of the light signal propagating through the optical fiber cable.

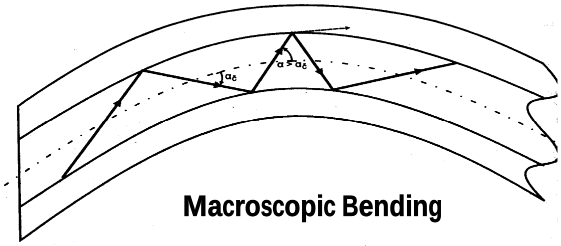
ii. Scattering loss: Light signal scattering can be thought of as the deflection of a ray from a straight path, for example by irregularities in the propagation medium, particles or in the interface between the two media. Irregularities and defects (which are produced when optical fibers are manufactured) are main causes for the scattering of light in unexpected directions.



iii. Bending losses: This loss induced by physical stress on the fiber. Bending loss is classified according to the bend radius of curvature:

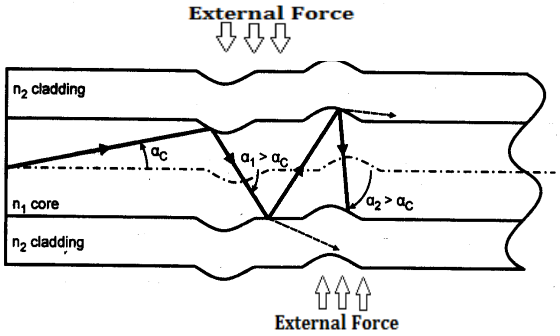
a) Macrobend Loss b) Microbend Loss

a) Macroscopic Bending: Macro-bend Losses are observed when a fiber bend's radius of curvature is large compared to the fiber diameter. These bends are a great source of loss when the radius of curvature is less than several centimeters.



Macroscopic Bending

b) Microscopic Bending: Micro-bend Loss are caused by small discontinuities or imperfections in the fiber. Uneven coating applications and improper cabling procedure increases micro bend loss. External forces are also a source of micro bends.



Microscopic Bending

APPLICATIONS OF OPTICAL FIBERS:

Due to its variety of advantages optical fiber has a wide range of application in different fields namely:

i. Communication:
ii. Medicine and
iii. Sensors etc.,

i. <u>COMMUNICATION</u>: Optical fibers are used as wave guides in the communication system. A typical block diagram of optical fiber communication system (OFCS) is shown in the following figure. It mainly consists of the following parts:

a) Encoder, b) Transmitter, c) Waveguide, d) Receiver and d) Decoder.

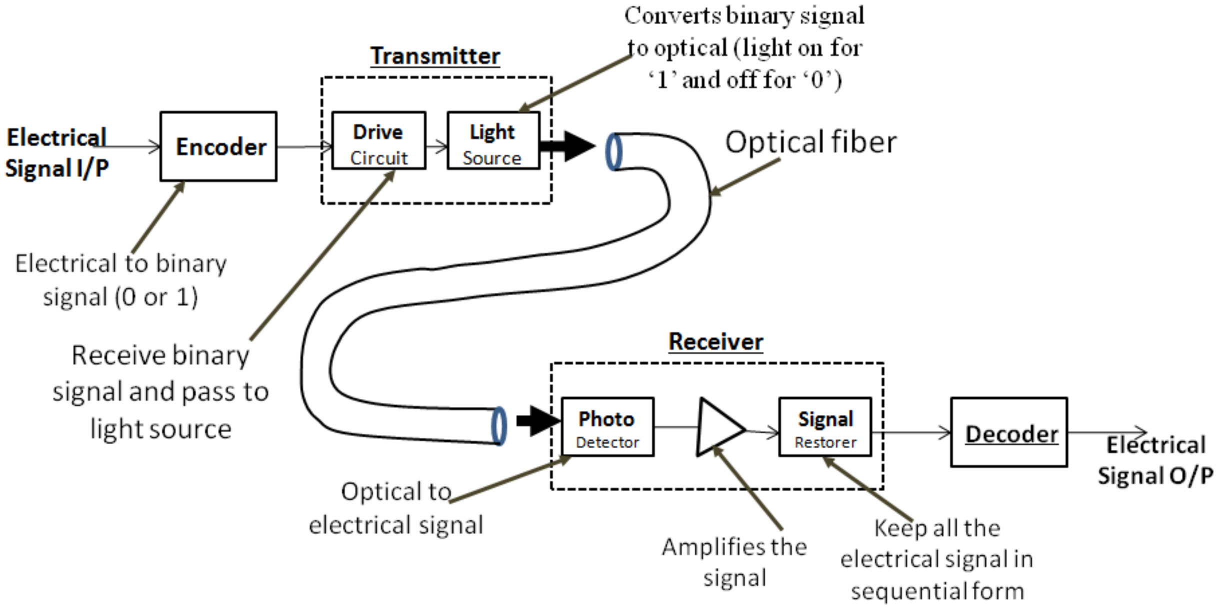


Fig.: Block Diagram of Optical Fiber Communication System

a) <u>Encoder</u>: The audio signal (i.e., the words spoken by us) is converted into electrical signal which is an analog signal. Encoder is an electronic circuit that converts this analog signal into binary or digital signals.

b) <u>Transmitter</u>: The digital signal from the encoder is fed to the transmitter which consists of two parts- Drive circuit and Light source. Drive circuit receives the digital signal from encoder and feeds it to the light source. Light source is usually LED or a Diode LASER. If digital '0' is received then light source will be turned OFF. If digital '1' is received then the light source will be turned ON. Thus light source converts electrical signals into optical signals.

c) <u>Waveguide</u>: Now the Optical signals generated by the transmitter are fed to an optical fiber which acts as waveguide. The signal traverses over longer distances through these waveguides.

d) <u>Receiver</u>: On the other side of the waveguide, he optical signal is received by the receiver which consists of Photo detector, amplifier and a signal restorer. The Photo detector receives the optical signal and generates the equivalent electrical signals. These electrical signals are amplified by the amplifier. The signal restorer keeps all the electrical signals in a sequential form and supplies to decoder.

e) **Decoder:** It is an electronic system that converts the digital signal to analog signal.

ii. MEDICAL:

Optical fibers are generally used in Endoscopy. They are also used in LASER Angioplasty (Laparoscopic Surgery) which is usually used for operations in the stomach area such as appendectomies. A LASER Angioplasty usually makes use of three channels (bundles) of optical fibers. Channel 1(One optical fiber cable) is used to observe where exactly the cholesterol deposits are present. LASER of suitable power is sent through channel 2 to destroy the cholesterol deposits. Channel 3 is used to suck out the debris.

iii. SENSORS:

Another important application of optical fibers is in sensors. If a fiber is stretched or squeezed, heated or cooled or subjected to some other change of environment, there is usually a small but measurable change in light transmission.

Level Sensors:

A chamgered Optical fiber, containing of core alone is used in a level sensor. The condition here is that the refractive indices of air, core and liquid should such that $n_{air} < n_{core} < n_{liquid}$.

A light signal from the source is fed to the fiber core as shown in fig. (i). The light signal reaches the detector after getting subjected to total internal reflections in the core. It is observed here that the liquid level did not touch the tip of the optical fiber yet and hence the light signal smoothly reaches the detector.

When the liquid raises to sufficient level as shown in figure (ii), at the point of core-liquid interface, total internal reflection cannot take place. The reason is $n_{core} < n_{liquid}$. Therefore the light signal gets leaked into liquid without reaching the detector.

Thus the light signal reaching the detector indicates lower liquid level while the detector does not receive the light signal, indicates sufficient level of the liquid.

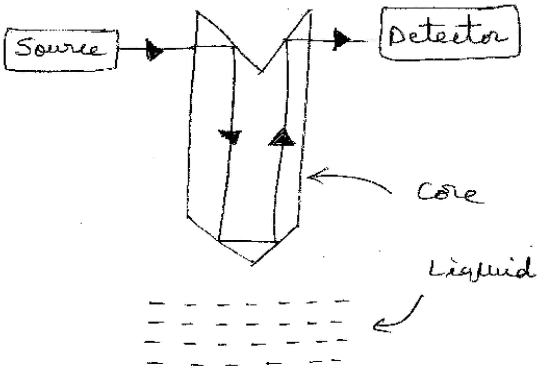


Fig (i)

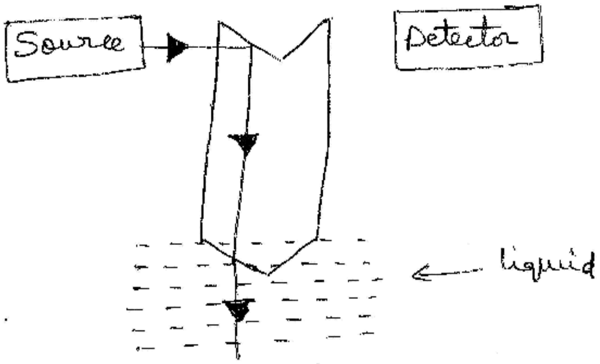


Fig (ii)

Electromagnetic Theory

Scalar: A physical quantity which has only magnitude is called Scalar.

Eg Mass, temperature etc

Vector: A physical quantity which has both magnitude and direction is called Vector.

Field: Field is a function that specifies the particular quantity everywhere in a region.

If the physical quantity is scalar then the field is said to be Scalar field.

Eg Temperature distribution in the building

If the physical quantity is vector then the field is said to be vector field.

Eg Electric field.

Scalar product:

Scalar product of two vectors is given by the product of magnitude of ~~one~~ first vector and the cosine component of the second vector in the direction of the first vector.

$$\vec{F}\cdot\vec{S} = |\vec{F}||S|\cos\theta.$$

The results of this product is a Scalar. This product is also called Dot product

(i) If the dot product is zero then the ~~two~~ vectors are perpendicular

(ii) Dot product is maximum when both the vectors are parallel.

$$\overline{A}\cdot\overline{B} = A_xB_x + A_yB_y + A_zB_z$$

Vector Product:

It is also called ~~vector~~ cross product. It is defined as. the product of magnitude of the first vector and the sine component of the second vector. The resultant is a vector and is directed perpendicular to both the ~~vecto~~ first and second vectors

parallel	zero
perpendicular	max

~~Rules~~

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Rules to be followed on dot and cross product:

If $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x, y and z axes respectively then.

$\hat{i}\cdot\hat{i} = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{i}\cdot\hat{k} = 0$

$\hat{j}\cdot\hat{k} = 0$

$\hat{j}\cdot\hat{i} = 0$

$\hat{k}\cdot\hat{i} = 0$

$\hat{k}\cdot\hat{j} = 0$

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{k}\times\hat{j} = -\hat{i}$

$\hat{i}\times\hat{k} = -\hat{j}$

Problem: (1) Determine $\bar{A}\cdot\bar{B}$ and $\bar{A}\times\bar{B}$

if $\bar{A} = 3\hat{j}+2\hat{k}$

$\bar{B} = 5\hat{i}+8\hat{j}$

$$\bar{A}\cdot\bar{B} = (3\hat{j}+2\hat{k})\cdot(5\hat{i}+8\hat{j})$$

$$= (3)(5)(\hat{j}\cdot\hat{i}) + (3)(8)(\hat{j}\cdot\hat{j}) + (2)(5)(\hat{k}\cdot\hat{i}) + (2)(8)(\hat{k}\cdot\hat{j}).$$

$$= 0+24(1)+0+0.$$

refer to rules in the previous page

$$\boxed{\bar{A}\cdot\bar{B} = \underline{\underline{24}}}$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 2 \\ 5 & 8 & 0 \end{vmatrix}$$

$$= \hat{i}(0-16) - \hat{j}(0-10) + \hat{k}(0-15)$$

$$\boxed{\bar{A}\times\bar{B} = -16\hat{i}+10\hat{j}-15\hat{k}}$$

(2) Evaluate the Scalar Triple product for

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\bar{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

$$\bar{B}\times\bar{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \hat{i}(B_yC_z - B_zC_y) - \hat{j}(B_xC_z - B_zC_x) + \hat{k}(B_xC_y - B_yC_x).$$

2(b)

$$\bar{A}\cdot(\bar{B}\times\bar{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x).$$

$$\Rightarrow \bar{A}\cdot(\bar{B}\times\bar{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

By simple manipulation of the term in the above two equations we can prove that

$$\bar{A}\cdot(\bar{B}\times\bar{C}) = \bar{B}\cdot(\bar{C}\times\bar{A}) = \bar{C}\cdot(\bar{A}\times\bar{B}).$$

Problem (3):

If $\bar{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$;
$\bar{B} = 3\hat{i} + 4\hat{j} + 2\hat{k}$ and
$\bar{C} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ show that

$$\bar{A}\times(\bar{B}\times\bar{C}) = (\bar{A}\cdot\bar{C})\bar{B} - (\bar{A}\cdot\bar{B})\bar{C}$$

LHS. $\bar{A}\times(\bar{B}\times\bar{C})$.

$$(\bar{B}\times\bar{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}\,(12-4) - \hat{j}\,(9-8) + \hat{k}\,(6-16).$$

$$= 8\hat{i} - \hat{j} - 10\hat{k}.$$

$$\bar{A}\times(\bar{B}\times\bar{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix}$$

$$= \hat{i}\,[-30-(-4)] - \hat{j}\,[-20-32] + \hat{k}\,(-2-24)$$

$$= -26\hat{i} + 52\hat{j} - 26\hat{k}.$$

RHS

$(\bar{A}\cdot\bar{C})\bar{B} - (\bar{A}\cdot\bar{B})\bar{C}$

$(\bar{A}\cdot\bar{C})\bar{B} = \left[(2\hat{i}+3\hat{j}+4\hat{k})\cdot(4\hat{i}+2\hat{j}+3\hat{k})\right] \; 3\hat{i}+4\hat{j}+2\hat{k}$

$= \cancel{8+6+}$

$= (8+6+12)(3\hat{i}+4\hat{j}+2\hat{k})$

$= (26)(3\hat{i}+4\hat{j}+2\hat{k})$

$= 78\hat{i}+104\hat{j}+52\hat{k}$

$(\bar{A}\cdot\bar{B})\bar{C} = \left[(2\hat{i}+3\hat{j}+4\hat{k})\cdot(3\hat{i}+4\hat{j}+2\hat{k})\right]\cdot(4\hat{i}+2\hat{j}+3\hat{k})$

$= (6+12+8)(4\hat{i}+2\hat{j}+3\hat{k})$

$= 26(4\hat{i}+2\hat{j}+3\hat{k})$

$= 104\hat{i}+52\hat{j}+78\hat{k}$

$(\bar{A}\cdot\bar{C})\bar{B} - (\bar{A}\cdot\bar{B})\bar{C} = (78\hat{i}+104\hat{j}+52\hat{k}) - (104\hat{i}+52\hat{j}+78\hat{k})$

$= -26\hat{i}+52\hat{j}-26\hat{k}$

$$\therefore \boxed{\bar{A}\times(\bar{B}\times\bar{C}) = (\bar{A}\cdot\bar{C})\bar{B} - (\bar{A}\cdot\bar{B})\bar{C}}$$

Problem (4):

$\bar{A} = 6\hat{j}+\hat{k}$

$\bar{B} = 2\hat{i}+4\hat{j}$

Find $\bar{A}\cdot\bar{B}$ and $\bar{A}\times\bar{B}$ and the angle between $\bar{A}$ and $\bar{B}$?

$\bar{A}\cdot\bar{B} = (6\hat{j}+\hat{k})\cdot(2\hat{i}+4\hat{j})$

$= 24$

$$\bar{A}\times\bar{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 6 & 1\\ 2 & 4 & 0\end{vmatrix}$$

$$= \hat{i}(0-4) - \hat{j}(0-2) + \hat{k}(0-12)$$

$$\bar{A}\times\bar{B} = -4\hat{i}+2\hat{j}-12\hat{k}$$

~~$\cos\theta = A$~~

$$\bar{A}\cdot\bar{B} = |A|\,|B|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\bar{A}\cdot\bar{B}}{|A|\,|B|} = \frac{24}{(\sqrt{36+1})(\sqrt{4+16})} = \frac{24}{\sqrt{37}\sqrt{20}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{24}{\sqrt{37}\sqrt{20}}\right)$$

$$\Rightarrow \boxed{\theta = 28.1^\circ}$$

Gradient of a Scalar ~~function~~ field:

~~∇~~ If temperature T is considered as scalar field then ∇T ~~gives the~~ is defined as gradient of temperature in a given direction, where ∇ is given by $\nabla = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$. Here $\vec{\nabla}$ is a vector operator, operating on a scalar field (T), resulting in a vector, ∇T.

DEL operator is a vector differential operator denoted by $\vec{\nabla}$. It can be operated on scalar and vector fields.

$$\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).$$

Gradient of a Scalar field:

operating del operator on a SCALAR field results in the GRADIENT. Gradient of a scalar field gives the maximum rate of change of the field at a point.

Let $T(x,y,z)$ denote the temperature in a room at the point (x,y,z). If we move a little bit distance from this point, the variation in temperature (dT) is given by.

$$dT = \left(\frac{\partial T}{\partial x}\right)dx + \left(\frac{\partial T}{\partial y}\right)dy + \left(\frac{\partial T}{\partial z}\right)dz \longrightarrow ①$$

The above equation can be rewritten as.

$$dT = \left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\boxed{dT = \nabla T \cdot dl} \longrightarrow ②$$

$(\text{where } dl = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

Here ∇T is the gradient of temperature (T) and it is a vector quantity

Geometrical interpretation of the Gradient:

Eq ② can be rewritten as.

~~$dT = |\nabla T|$~~

$$dT = \nabla T \cdot dl = |\nabla T|\,|dl| \cos\theta \longrightarrow ③$$

where (θ) is the angle between (∇T) and (dl).

From the above equation it is clear that maximum change in (T) evidentally occurs when $\theta = 0$; that is for a fixed distance $|dl|$, dT is greatest when ~~I~~ we move in the same direction as ∇T. Thus

(i) The gradient ∇T points in the direction of maximum increase of the function (T).

(ii) The magnitude $|\nabla T|$ gives the slope along this maximal direction.

<u>Note</u>: $\nabla(V+U) = \nabla V + \nabla U$;

$\nabla(VU) = V\nabla U + U\nabla V$

$\nabla\left(\frac{V}{U}\right) = \frac{U\nabla V - V\nabla U}{U^2}$; $\nabla V^n = nV^{n-1}\nabla V$.

<u>Problem:</u>

The potential function of a scalar field is $V = 7x + 3y - 2z$. Find the gradient.

Given $V = 7x + 3y - 2z$

$$\nabla V = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(7x+3y-2z).$$

$$= \hat{i}\,7 +$$

$$\boxed{\nabla V = 7\hat{i} + 3\hat{j} - 2\hat{k}}$$

<u>Divergence of a vector field:</u>

The dot product of ∇ operator on a vector field results in the Divergence. It is a scalar.

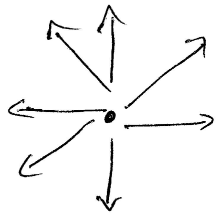
Let $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$ then.

$$\vec{\nabla}\cdot\vec{V} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}V_x + \hat{j}V_y + \hat{k}V_z\right)$$

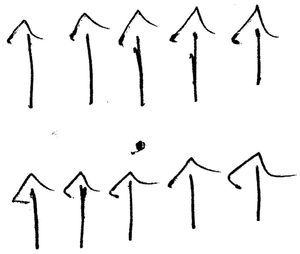
$$\boxed{\nabla\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}.} \rightarrow ①$$

<u>Geometrical interpretation:</u>

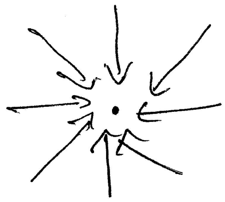
(i) $\nabla\cdot\vec{V}$ is a measure of how much the vector (V) spreads out (diverges) from the point in question. So it can be defined as the outward flux per unit volume as the volume shrinks about the point.



Positive divergence



zero divergence



negative divergence

(ii) Divergence is positive at a source point in the field and negative at a sink point. It is Zero where there is neither sink nor source

(iii) In a fluid while it is flowing, if divergence is positive at any point then the fluid expands and the density at that point decreases. If divergence is negative then the fluid contracts and the density of the fluid at that point increases.

If the divergence is zero then the density of the fluid remains same all along its flow. Such fluids are called Incompressible fluids.

(iii) If $\vec{J}$ is electric current density vector, then $\nabla\cdot\vec{J}$ is the divergence of electric current density vector, which gives the charge flowing out of unit volume.

Problem: Determine the divergence of the vector field

$$P = x^2yz\,\hat{i} + xz\,\hat{k}.$$

$$\vec{\nabla}\cdot\vec{P} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(x^2yz\,\hat{i} + xz\,\hat{k}\right)$$

$$= \underline{2xyz + x.}$$

Note: $\nabla\cdot(\vec{V}+\vec{U}) = \nabla\cdot\vec{V} + \nabla\cdot\vec{U}$

$\nabla\cdot(k\vec{V}) = \cancel{k\nabla\cdot V}\; k\nabla\cdot\vec{V} + \vec{V}\cdot\nabla k$

where k is a scalar

curl of a vector field:

The cross product of ∇ operator on a vector field result in the curl of the vector field. It is a vector.

Let $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$ then

$$\nabla \times \vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

Geometrical interpretation (Physical significance):

(i) The curl of a velocity vector results in angular velocity. Therefore the curl of vector represents the rotation sense of a vector field. If $\nabla\times\vec{V}$ is equal to zero then the vector is called Irrotational vector.

(ii) ~~$\nabla\times(\vec{A}\times\vec{B}) = (\nabla\times\vec{A})+(\vec{\nabla}\times\vec{B})$~~

~~$\nabla\times(\vec{A}\times\vec{B}) = \vec{B}(\nabla\times\vec{A}) - \vec{A}.\nabla.$~~

(ii) $\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$ ie; the curl value of electric field vector is equal to rate of change of magnetic flux density vector

~~(iii) $\nabla\times B = \frac{\partial \vec{J}}{\partial t}$. ie, the curl of mag~~

Problem: If $\vec{A} = 2xz^2\hat{i} - yz\hat{j} + 3xz^3\hat{k}$ then determine curl $\vec{A}$ at the point (1,1,1).

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xz^2 & -yz & 3xz^3 \end{vmatrix}$$

$$\Rightarrow \quad \nabla \times \vec{A} = \hat{i}\left[\frac{\partial}{\partial y} 3xz^3 - \frac{\partial}{\partial z}(-yz)\right] - \hat{j}\left[\frac{\partial}{\partial x} 3xz^3 - \frac{\partial}{\partial z} 2xz^2\right] + \hat{k}\left[\frac{\partial}{\partial x}(-yz) - \frac{\partial}{\partial y} 2xz^2\right].$$

$$= \hat{i}\,(0+y) - \hat{j}\,(3z^3 - 4xz) + \hat{k}\,(0-0).$$

$$\vec{\nabla} \times \vec{A} = y\,\hat{i} + (4xz - 3z^3)\,\hat{j}$$

at the point (1,1,1)

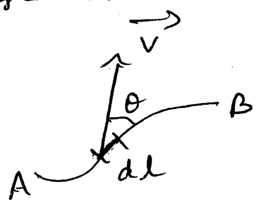
$$\boxed{\vec{\nabla} \times \vec{A} = \hat{i} + \hat{j}}.$$

Problem: Prove that $\nabla \cdot (\nabla \times A) = 0$.
where $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$.

Prove that curl grad $\phi = 0$.
where grad $\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$.

Line integral:

Line integration is the integration to be carried out along a line. In a vector field, AB is a curve. Let (dl) be an infinitesimal element, making an angle (θ) with the vector ($\vec{V}$).



$$\int_A^B \vec{V} \cdot \vec{dl} = \int_A^B V\,dl\cos\theta$$ is the line integral value along the path AB.

$$\int_A^B \vec{V} \cdot \vec{dl} = \int_A^B (V_x \hat{i} + V_y \hat{j} + V_z \hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int_A^B V_x\,dx + V_y\,dy + V_z\,dz$$

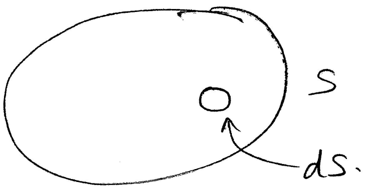
Physical Significance:

(i) A particle moving along the path AB, under the force $\vec{F}$. Then the work done can be calculated with the line integral $\int_A^B \vec{F}\cdot d\vec{l}$

(ii) Line integral $\int_A^B \vec{E}\cdot d\vec{l}$ results in the work done on a charged particle in an electric field. This is equal to the potential difference between (A) and (B).

Surface ~~and~~ Integral:

Consider a surface 'S' in a vector field. Let (ds) be an infinitesimal area.



If (E) is the electric field vector then $\vec{E}\cdot d\vec{s}$ gives the flux coming out of the infinitesimal area $d\vec{s}$.
Then the total flux emanating from the entire surface is given by

$$\iint \vec{E}\cdot d\vec{s}$$

Vector integral:

Volume integral over a volume can be written as

$\iiint \vec{A}\, dV$. where $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.

In the Cartesian co ordinates

$$\iiint \vec{A}\, dx\,dy\,dz = \iiint (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\, dx\,dy\,dz.$$

$$= \iiint A_x\hat{i}\, dx\,dy\,dz +$$

$$\iiint A_y\hat{j}\, dx\,dy\,dz +$$

$$\iiint A_z\hat{k}\, dx\,dy\,dz.$$

Stoke's Theorem and Gauss divergence theorem:

Stoke's theorem:

$$\oint \vec{A}\cdot d\vec{l} = \iint (\vec{\nabla}\times\vec{A})\cdot d\vec{s}.$$

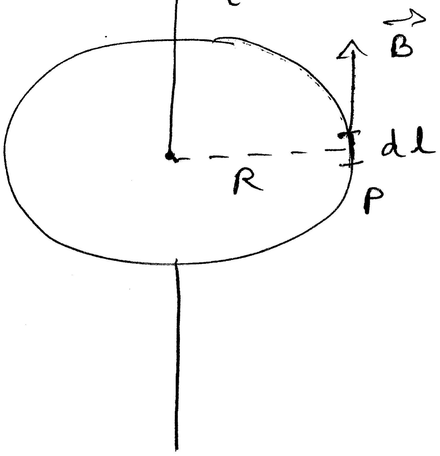
Line integral can be converted into surface integral.

Gauss Divergence theorem

$$\oiint \vec{A}\cdot d\vec{s} = \iiint (\vec{\nabla}\cdot\vec{A})\, dV$$

Ampere's law:

Oersted found that magnetic compass needle ~~always~~ shows deflection when it is placed near a current carrying conductor. Biot and Savart found that a magnetic pole experiences a force when placed near a current carrying conductor. Their experiments helped to evaluate a relation between the magnetic field (B) at any point in the space and the current flowing through the conductor. This relation is called Biot - ~~Savartt~~ Savart law.



~~Let (R) be a b closed loop of~~

Let (R) be the radius of the closed loop. Let (i) be the current flowing through the ~~g~~ closed loop, in a conductor. Then Ampere's law in states that 'the work done in moving a unit North pole around a closed loop is equal to μ_0 times the current flowing through the closed loop.'

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i$$

The magnetic flux density $\vec{B}$ at a point (P) located at a distance (R) from a thin uniform wire carrying current (i) is given by.

$$B = \frac{\mu_0 i}{2\pi R}$$ [from Biot Savart law].

According to Ampere's law.

$$\oint \vec{B}\cdot d\vec{l} = B\oint d\vec{l}$$

$$= \frac{\mu_0 i}{2\pi R}\oint dl.$$

The line integral of the closed loop is its circumference

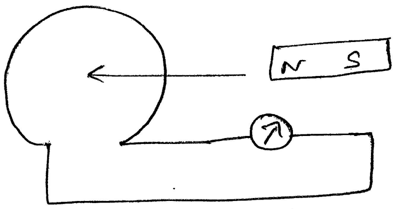
Hence $\oint dl = 2\pi R$.

$$\therefore \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 i}{2\pi R}\left[2\pi R\right] = \underline{\underline{\mu_0 i}}.$$

Faraday's law of electromagnetic induction:

'The induced emf in a closed electric circuit is equal to the negative rate of change of the magnetic flux through the surface bounded by the circuit.'

Faraday's experiment:



(i) Faraday connected a Galvanometer in series with a coil of wire (closed loop) as shown in the adjacent figure. (ii) When a bar magnet (N–S) was moved into the coil, galvanometer (G) showed a deflection in clock-wise direction. ~~If the bar magnet is drawn~~ (iii) when the bar magnet was drawn out of the loop, the needle of the galvanometer deflected in anti clock wise direction. ~~when the~~

(iv) when the bar magnet was kept stationary, the galvanometer ~~bar~~ did not show any deflection.

(V) This process is known as Electromagnetic induction. The current in the loop or through the galvanometer is called Induced current. The work done in moving a unit charge in the induced current around the loop is known as Induced electro motive force.

(Vi) It should be noted here that, even if the magnet is stationary and the coil is moved back and forth, the galvanometer shows similar deflections as mentioned above. This clearly shows that relative motion between the loop and magnet is necessary to induce current in the loop.

Faraday's first law:

When the magnetic flux linked with a closed circuit or a coil is changed in any manner, an electromotive force (emf) is induced in the circuit.

Faraday's second law:

The induced electro motive force in a circuit is directly proportional to rate of change of the magnetic flux (ϕ_B) linked with the circuit.

$$\mathcal{E} = -\frac{d\phi_B}{dt} \rightarrow ①$$

The negative sign indicates that the direction of induced emf is opposite to the change in magnetic flux.

Vector form of Faraday's law:

The magnetic flux (ϕ_B) is defined as integral of dot product of magnetic flux density ($\bar{B}$) and elemental surface area ($d\bar{s}$)

$$\therefore \phi_B = \oint_s \bar{B}\cdot d\bar{s} \rightarrow ②$$

Similarly the potential (V) is defined as the negative integral of dot product of electric field intensity $\vec{E}$ and elemental displacement vector $d\bar{l}$.

$$V = -\oint \bar{E}\cdot d\bar{l} \rightarrow ③$$

But induced emf, $\mathcal{E} = -V$

$$\therefore \mathcal{E} = \oint \bar{E}\cdot d\bar{l}$$

$$\therefore \mathcal{E} = \oint \bar{E}\cdot dl = -\frac{d}{dt}\oint \bar{B}\cdot ds \quad \text{using ①, ② and ③.}$$

From stoke's theorem,

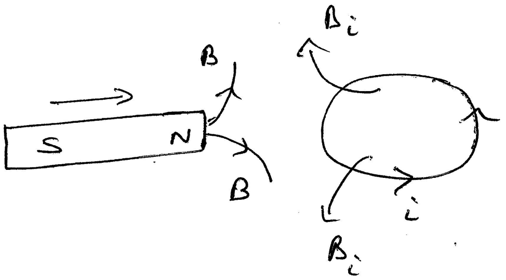
$$\oint \bar{E}\cdot d\bar{l} = \oint_s (\nabla\times\bar{E})\cdot d\bar{s}$$

$$\oint_s (\nabla\times\bar{E})\cdot d\bar{s} = -\frac{d}{dt}\oint \bar{B}\cdot d\bar{s}$$

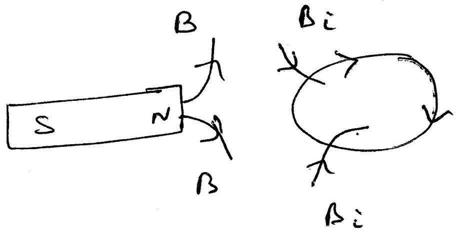
$$\therefore \boxed{\nabla\times E = \frac{\partial \bar{B}}{\partial t}}$$

Lenz's law:

In 1834 Henry Federick Lenz devised a law to find the direction of induced current in a closed circuit. The direction of induced current in a closed conducting loop is such that it always opposes the original cause that produced it. This is called Lenz's law. This is the reason why we have a negative sign in Faraday's law.



Consider a bar magnet and a closed conducting loop. Let (B) be the magnetic flux density of the bar magnet. When the bar magnet is moved towards the closed loop, as shown in the adjacent figure, a current is induced in the anti clockwise direction in the loop and the face of the loop towards the north pole of the bar magnet behaves as north pole. The induced magnetic flux density (B_i), thus tries to repel the North pole of the bar magnet, approaching the closed loop.



When the bar magnet is moved away from the closed loop as shown in the adjecent figure, a current is induced in the clockwise direction in the loop and the face of the loop towards the north pole of the bar magnet behaves as SOUTH POLE. The induced magnetic flux density (B_i), thus tries to attract the ~~Sout~~ North pole of the bar magnet, RECEDING away from the closed loop.

Maxwell equations in integral form:

(i) Gauss law in Electrostatics:

Consider a point charge ~~(q)~~ (dv) in a closed surface. Then the flux ($d\phi_E$) because of this point charge coming out of an ~~etc~~ infinitesimal area (ds) is given by.

$$d\phi_E = \vec{E}\cdot d\vec{s}$$

According to Gauss law of electrostatics the total flux (ϕ_E) coming out of any closed surface is proportional to the total charge (Q) contained in it and is equal to $\left(\frac{1}{\epsilon_0}\right)$ times the total charge (Q) i.e;

$$\boxed{\phi_E = \int d\phi_E = \oint \vec{E}\cdot d\vec{s} = \frac{Q}{\epsilon_0}} \longrightarrow ①$$

(ii) Gauss law in Magnetostatics:

~~Et~~ Let (ϕ_B) be the total magnetic flux because of all magnets ~~and~~ in a closed surface. Then according to Gauss law of magnetostatics the total magnetic flux (ϕ_B) enclosed by a surface is ZERO.

$$\phi_B = \oint \vec{B}\cdot d\vec{s} = 0.$$

This law clearly states that there are no isolated poles. Magnetic poles always exist in pair - North and South. The magnetic lines of force coming out of the North pole will again enter the South pole. Thus when a closed surface is considered, the amount of flux leaving the surface is equal to the amount of flux entering it. Hence the net flux or total flux is ZERO always.

(iii) Faraday's law of Electromagnetic induction:

When a bar magnet is moved either towards or away from a closed conducting loop, an emf ~~called~~ is induced in the conducting loop that depends on the ~~sad~~ negative rate of change of magnetic flux. This induced emf ($\mathcal{E}$) is given by,

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}$$

but $\phi_B = \oint \vec{B} \cdot d\vec{s}$

$$\boxed{\therefore \mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{s}} \longrightarrow ③$$

This is known as integral form of Faraday's law.

(iv) Ampere's law:

The work done in moving a unit magnetic North pole around any arbitrary shaped closed loop is equal to (μ_0) times the flow of current through the loop.

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 i} \longrightarrow ④$$

~~This~~

Note ~~B=~~, $\mu_0 = \frac{B}{H}$ or $H = \frac{B}{\mu_0}$.

$\therefore$ eq ④ can be written as $\oint \vec{H} \cdot d\vec{l} = i$.

Maxwell equations in differential form:

(i) Gauss law in Electrostatics:

The integral form of Gauss law in Electrostatics is

$$\boxed{\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}} \longrightarrow ①.$$

If (dq) is the charge enclosed in an elemental volume (dV) then the volume charge density (ρ) is given by

$$\rho = \frac{dq}{dV}$$

Then the total charge enclosed by the volume is given by

$$Q = \int dq = \int_V \rho\, dV$$

$$\Rightarrow \boxed{\oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int_V \rho\, dV.} \longrightarrow ②.$$

From Gauss divergence theorem,

$$\oint \vec{E}\cdot d\vec{s} = \int_V (\nabla\cdot E)\, dV.$$

Using this equation in ②, we get

$$\oint \vec{E}\cdot d\vec{s} = \int (\nabla\cdot E)\, dV = \frac{1}{\varepsilon_0}\int_V \rho\, dV.$$

$$\Rightarrow \varepsilon_0 \int_V (\nabla\cdot E)\, dV = \int_V \rho\, dV$$

$$\Rightarrow \int_V (\nabla\cdot \varepsilon_0 E)\, dV = \int_V \rho\, dV$$

$$\Rightarrow \int_V (\nabla\cdot D)\, dV = \int_V \rho\, dV. \quad [\because D = \varepsilon_0 E].$$

D - Electric displacement vector or Electric flux density or Displacement current.

$$\Rightarrow \boxed{\nabla\cdot D = \rho.}$$

This is called the Point form ~~of~~ or differential form of Gauss law in electrostatics

$\because D = \varepsilon_0 E$, $\nabla\cdot D = \rho \Rightarrow \nabla\cdot E = \frac{\rho}{\varepsilon_0}$ is another form.

(ii) Gauss law in Magnetostatics:

The integral form of Gauss law in Magnetostatics is $\phi_B = \boxed{\oint \vec{B}\cdot d\vec{s} = 0.} \longrightarrow ①$.

From Gauss divergence theorem, Surface integral can be converted into volume integral

$$\oint \vec{B}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{B})\, dV$$

$$\therefore \phi_B = \oint \vec{B}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{B})\, dV = 0.$$

$$\Rightarrow \int_V (\nabla\cdot B)\, dV = 0.$$

As the volume under consideration is arbitrary

$$\boxed{\nabla\cdot B = 0.}$$

This is called the Point form or differential form of Gauss law in Magnetostatics.

(iii) Faraday's law in electromagnetic induction:

The integral form of Faraday's law in electromagnetic induction is

~~$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot d\vec{s}$~~

$$\boxed{\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = \frac{-\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}} \longrightarrow ①.$$

According to Stokes theorem,

$$\oint \vec{E}\cdot dl = \int_S (\nabla\times\vec{E})\cdot d\vec{s}$$

Using this equation in eq①, we get

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = \int_S (\nabla\times\vec{E})\cdot d\vec{s} = \frac{-\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}.$$

$$\Rightarrow \int_S (\nabla\times\vec{E})\cdot d\vec{s} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$$

$$\Rightarrow \int_S (\nabla\times\vec{E})\cdot d\vec{s} = -\int_S \left(\frac{\partial\vec{B}}{\partial t}\right)\cdot d\vec{s}.$$

$$\Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

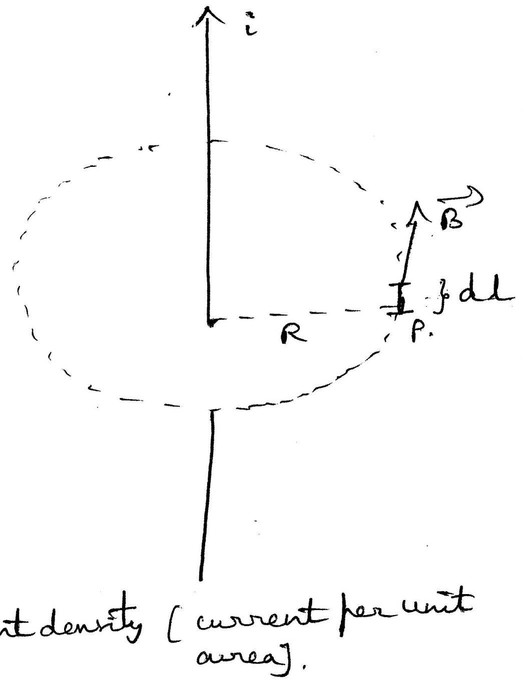
This is the point form or differential form of Faraday's law in Electro magnetic induction.

Ampere's law:

The integral form of Ampere's law is

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 i \longrightarrow ①.$$

where (i) is the current flowing the surface enclosed by the loop.



From stokes theorem,

$$\oint\vec{B}\cdot d\vec{l} = \int_S(\nabla\times\vec{B})\cdot d\vec{s}$$

Using this equation in ①, we get

$$\oint\vec{B}\cdot d\vec{l} = \int_S(\nabla\times\vec{B})\cdot d\vec{s} = \mu_0 i. \quad \hookrightarrow ②.$$

If (di) is the current through the elemental surface area (ds) then

$$di = \vec{J}\cdot d\vec{s}$$

where $\vec{J}$ is called current density [current per unit area].

$$\Rightarrow i = \int di = \int_S \vec{J}\cdot d\vec{s}$$

Using this eq in eq ②, we get

$$\int(\nabla\times\vec{B})\cdot d\vec{s} = \mu_0\int_S\vec{J}\cdot d\vec{s} = \int_S(\mu_0\vec{J})\cdot d\vec{s}.$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

This equation is called Point form or differential form of Ampere's law and is applicable for steady currents only. It is not applicable for time varying currents.

Maxwell's correction to Ampere's law - Displacement current

(or)

~~Modified Ampere's law~~

Ampere's modified law

(or)

Ampere - Maxwell law:

We know that current in a conductor produces magnetic field. Maxwell proved that a changing electric field in vacuum or in dielectric also produces a magnetic field. So, a changing electric field is equivalent to a current which flows as long as the electric field is changing and produces the same magnetic effects as an ordinary current. This is known as Displacement current (D).

Ampere's law in point form is

$$\nabla \times \vec{B} = \mu_0 \vec{J} \rightarrow ①$$

Taking divergence on both sides, we get

$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J})$$

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 (\nabla \cdot \vec{J})$$

$$\text{But } \nabla \cdot (\nabla \times \vec{B}) = 0 \quad \left[\because \text{divergence of curl of a vector is zero}\right]$$

$$\Rightarrow \boxed{\nabla \cdot \vec{J} = 0} \rightarrow ②$$

This equation contradicts with the 'equation of continuity', according to which

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \text{ or}$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Therefore Maxwell concluded that eq ① is incomplete. He suggested that 'something' must be added to $\vec{J}$ in eq ① such that the divergence of both sides is same.

Thus $\nabla \times \vec{B} = \mu_0 \vec{J} +$ Something $\rightarrow$ ③.

In order to know this 'something', Maxwell postulated that similar to the electric field due to changing magnetic field (Faraday's law of induction) there would be a magnetic field due to the changing electric field and he brought into the picture - Displacement current (D).

From the Point form of Gauss law, we have

$$\nabla \cdot \vec{D} = \rho$$

$$\Rightarrow \nabla \cdot \frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho}{\partial t}$$

Adding $\nabla \cdot \vec{J}$ on both side, we get

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = \nabla \cdot \vec{J} + \nabla \cdot \frac{\partial \vec{D}}{\partial t}.$$

$$= \nabla \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

According to equation of continuity

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = \nabla \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0.$$

Thus $\nabla \cdot \vec{J} = 0$ for steady current

and

$\nabla \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0$ for steady current as well as current corresponding to time varying fields.

Thus Ampere's law becomes.

$$\boxed{\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right).}$$

This is called Ampere's modified law or Maxwell - Ampere law.

Wave equation:

Consider a homogeneous, isotrophic and dielectric medium such as free space (or Vacuum). This medium offers infinite resistance to flow of charge and hence its conductivity is zero. As there are no free charges, the charge density is zero. Also the current density $\vec{J}$ is zero.

Thus the Maxwell equations become

(i) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \nabla \cdot E = 0 \quad (\because \rho = 0)$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \left[\because \vec{J} = 0 \right]$

Applying curl on both sides on eq (iv), we get

$$\nabla \times \nabla \times \vec{B} = \nabla \times \left[\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

$$\Rightarrow \nabla \times \nabla \times \vec{B} = \cancel{\nabla} \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left[\nabla \times E \right].$$

We know that

$$\nabla \times \nabla \times \vec{B} = \nabla (\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$$

$$\therefore \nabla (\nabla \cdot B) - \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left[\nabla \times \vec{E} \right].$$

from (ii) $\nabla \cdot \vec{B} = 0$

from (iii) $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

$$\therefore -\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left[-\frac{\partial \vec{B}}{\partial t} \right]$$

$$\Rightarrow \boxed{\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}.} \rightarrow ①$$

$D = \epsilon_0 E \Rightarrow \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t}$ in eq (iv).

Similarly by applying curl on eq(iii), we get

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \longrightarrow ②$$

Equations ① and ② represent the time varying electric and magnetic fields in free space. These equations are similar to the standard wave equation

$$\boxed{\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \longrightarrow ③$$

Comparing eq ①, ② and ③, we get

$$\frac{1}{v^2} = \mu_0 \varepsilon_0$$

$$\Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

Substituting the value of permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ and permitivity of free space $\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$ units

$$\text{we get } v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^9}}}$$

$$= \frac{1}{\sqrt{\frac{1}{9} \times 10^{-16}}} = \frac{1}{\frac{1}{3} \times 10^{-8}}$$

$v = 3 \times 10^8$ m/second. ie; velocity of light.

Thus electric and magnetic components $\vec{E}$ and $\vec{B}$ spread in the space with a velocity (v) which is equal to the velocity of light (C).